Biomaterials Engineering and Prosthesis Branch

Fourth Class

First Course

(2023-2024)

Principles of Composite Materials

References:

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2) Autar K. Kaw, "Mechanics of Composite Materials" second edition, New York , (2006).
3) Derek. Hull," An Introduction to Composite Materials", Cambrige University, (1995).
4) Deborch D.L. Chung, "Composite Materials, Science and Applications", Second edition, Springer, (2010).
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Composite Materials

1- Introduction to Composite Materials:

1.1- Definitions, Properties and Applications

A Composite material is formed when two or more materials are combined on a macroscopic scale, so that the properties of composite are different (usually better) from those of the individual constituents.

Properties that can be improved by composite materials

- Strength
- stiffness
- fatigue life
- corrosion resistance
- high temperature performance
- wear resistance
- thermal insulation
- thermal conductivity
- a acoustical insulation
- weight
- hardness
- etc.

Composite materials have a wide range of practical application in the industry like:

- Car body manufacturing
- Air plane structure
- Space land and space satellite
- Boats manufacturing
- Storage containers
- House appliances

The important property that recognizes the composite material on metal is the strength to the density ratio or strength to weight ratio.

Composite are made of
- → Continuous medium "matrix"
- → Discontinuous medium "Reinforcement" (Which is usually harder and stronger one)

Therefore, the properties of composite are depending on the properties of the matrix and reinforcement materials, their distribution and interaction.

Matrix: It is the material that work to bind the reinforcing material together in order to make a composite material that can carry loads or stresses.

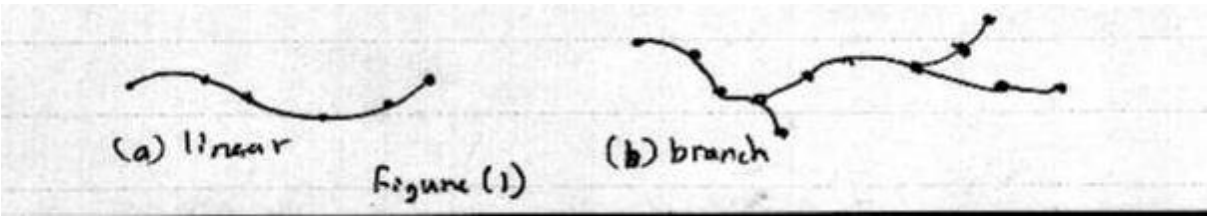
It is also called "**medium** ", it is may be **metal, polymer or ceramic**.

The polymers are most widely used as a "matrix". And are also called resins.

Polymers are very complex organic compounds whose molecular weight exceed 5000

Polymers have low electrical and thermal conductivity, therefore use for electrical and thermal insulation.

The molecules of polymers may have the linear, branch (fig.1) or three dimensional (spatial) structure repeated many times.



Polymers may be classified as:

1- **Thermoplastic resins**: for examples: nylon and polyethylene.

Soften when heated and become hard again when the heat is removed. And have linear chains or branch chains for their structure.

2- **Thermosetting resins**: for example: epoxy and polyester.

Do not soften when heated, but char and decompose and have cross linked structure.

3- **Elastomers resin**: - for example: rubber.

Is a polymer having considerable extensions and reversible. It is chains have some degree of cross- linking.

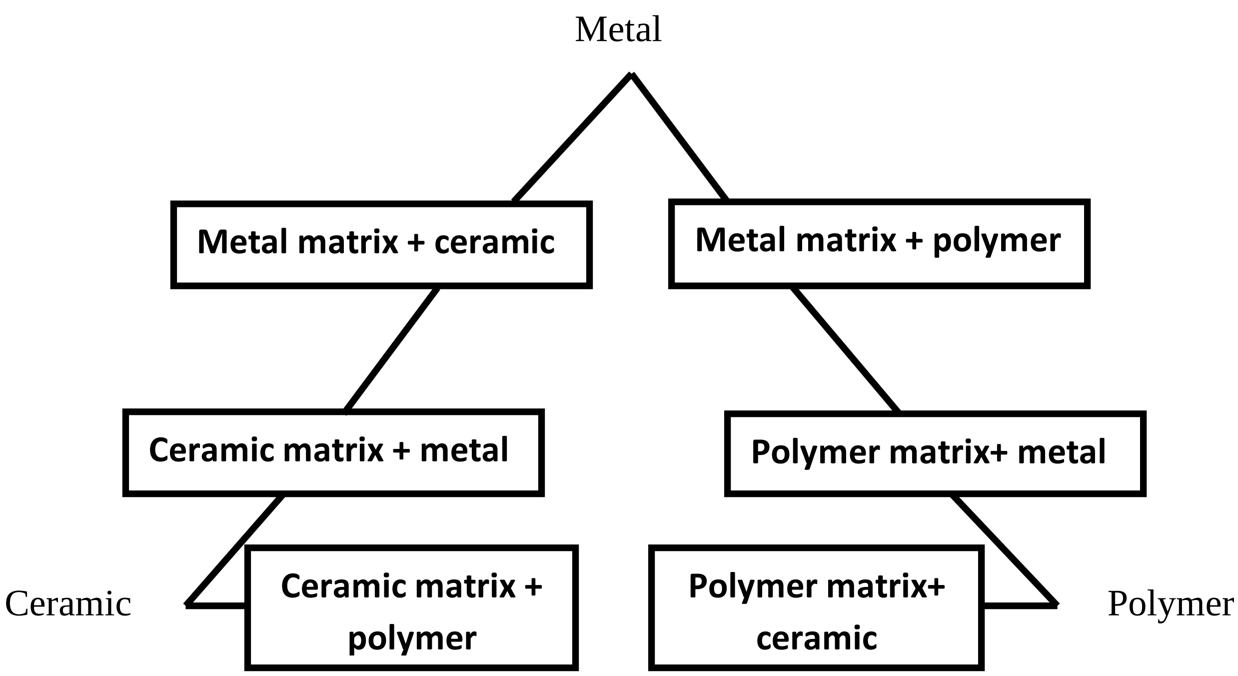
Reinforcing materials: It is the materials that make the reinforcing to the matrix. It is have different form may be fibers, particles, flakes, fillers and woven made from glass, carbon, Kevlar or steel…….etc.

1.2- Classification of Composite Materials

The diagram of the composite materials illustrated as following: -

Metal matrix + Metal

- Have high density.
- Thermal and electrical conductivity.
- Good strength, ductility and toughness.
- Working at medium and high temperature.
- Low corrosive resistance.

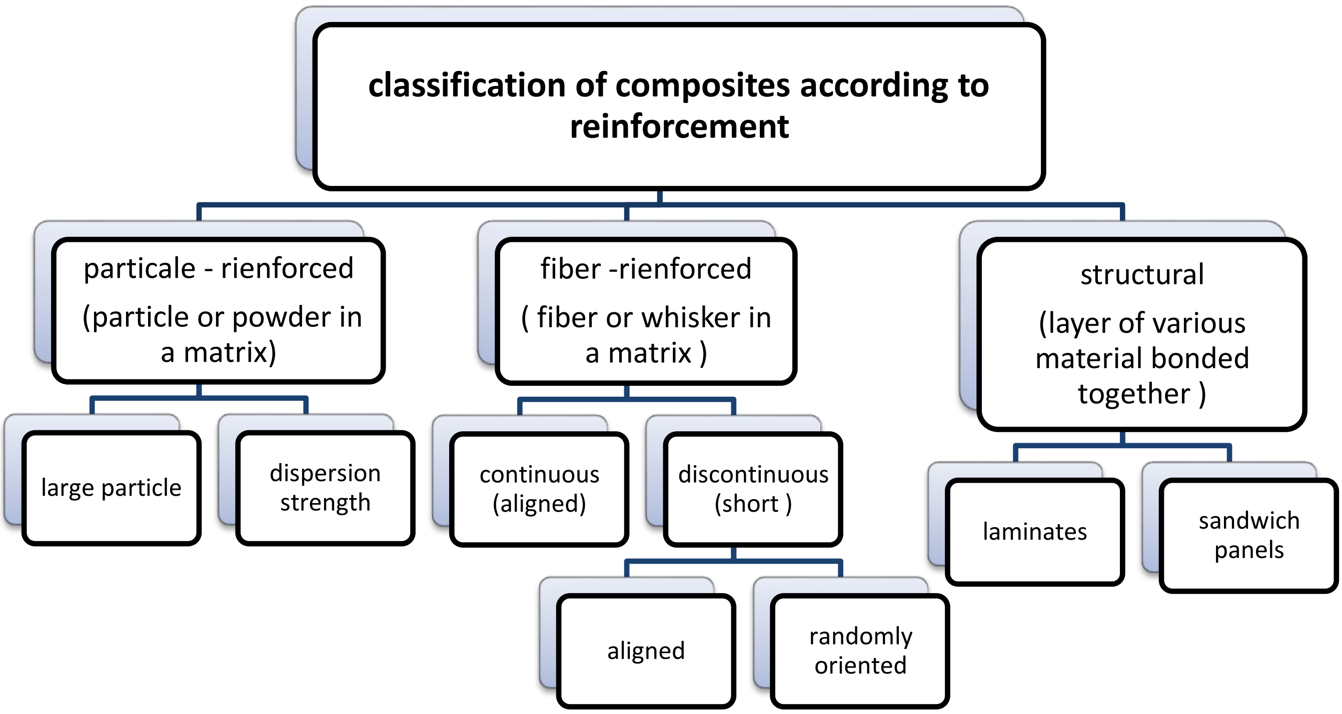


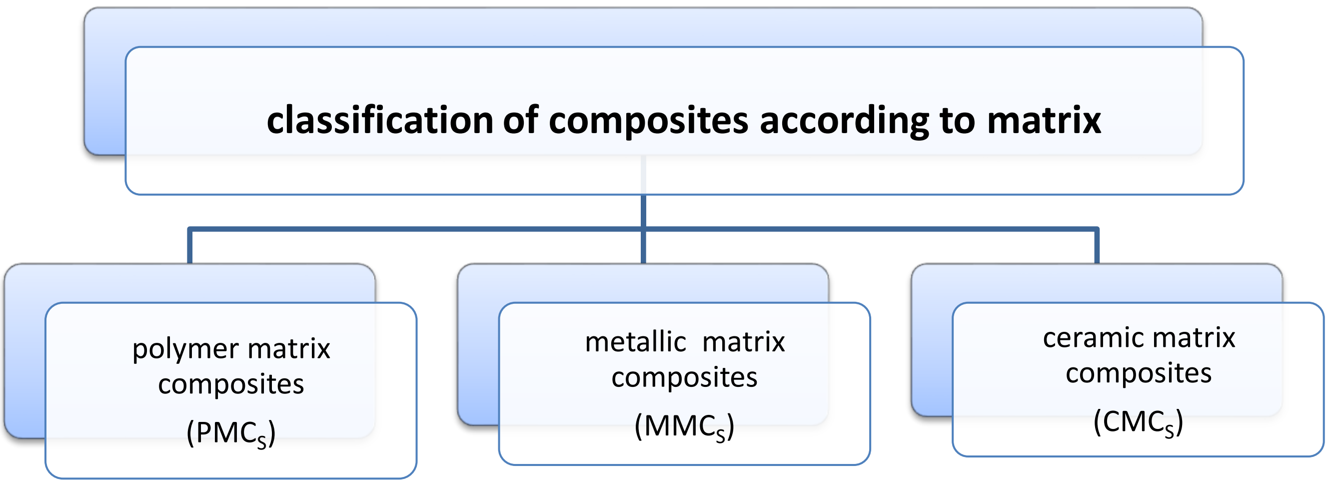
- Medium density.
- Electrical and thermal insulating.
-wear and corrosive resistance.
-difficult to form/machine.
-working at high temperature.

- Low density.
- High strength to weight ratio.
-have high corrosion resistance.
-easy to form.
-working at low temperature.

Ceramic matrix + Ceramic

Polymer matrix + Polymer





2.1- Composite Strengthened

2.1.1- Particulate Composite: - Consist of one or more materials suspended in a matrix of another material. The particles can be either metallic or non-metallic.

Particulate composite classified as:
- Large particles reinforced composite
- Dispersion strength composite

A classic example of polymers as a particulate composite material is carbon black in rubber (in manufacturing of tires). A carbon black improves strength, stiffness, wears resistance.

2.1.1.1- Large particle reinforced composite: Have particles with diameter of (1µm) or more and volume concentration (25-50) % or more of the composite.

One of their applications cermet or (cemented carbides), composite involving ceramic particles in a metal matrix which are widely used for the tips of cutting tools.

(e.g.)

Ceramic (Tungsten carbides)
[hard, brittle]

Metal
[soft, ductile]

→ Composite
[strong, hard, tough]

2.1.1.2- Dispersion strengthened composite: The strength of a metal can be increased by small particles dispersed throughout the matrix. The diameter of particle (0.1µm) and volume concentration (1-15) % of the composite. For example, the dispersion of aluminum – copper compound throughout of alloy. To product composite for general application, like piston, connecting rod for automotive application.

Also one way of introducing a dispersion of small particles throughout a metal uses sintering. Like dispersion of aluminum oxide (AL_2O_3) about (14%) throughout an aluminum matrix.

Where the tensile strength is
- 400 MPa for sintered aluminum powder
- 90 MPa for aluminum

The following table gives an example and applications of selected dispersion strengthened composites:

Composite	Applications
Ag - CdO	electrical contact material
Al - Al_2O_3	nuclear reaction
Pb - PbS	battery grids

The General Applications of Particulate Composite Materials

1- Non-metallic in non-metallic
- *Concrete*
- *Flakes glass in plastic matrix – for electrical insulating*

2- Metallic in non-metallic
- *Silver flakes in paint – for good conductivity*
- *Aluminum flakes in paint – aluminum paint for surface protection*

3- Non-metallic in metallic composites
- *Cermet are examples of ceramic and metallic*

4- Metallic in metallic composites
- *Lead particles in copper alloys to improve machinability*

Rule of mixtures

The rule of mixtures can predict the properties of the particulate composite material because the particulate composite materials depend only on relative amount and properties of the individual constituents.

$$\rho_c = \sum V_i * \rho_i = V_1 * \rho_1 + V_2 * \rho_2 + V_3 * \rho_3 + \cdots V_n * \rho_n$$

Where:

ρ_c= density of the composite

$\rho_1 \quad \rho_2 \;, \ldots. , \rho_n$: are the density of each constituent

$V_1 \, V_2 \quad \ldots \ldots \, V_n$: are the volume of fractions of each constituent

$$V_i = \frac{v_i}{v_c}$$

Where v_i: volume of the item (matrix or reinforcement).

v_c: total volume of the composite.

$$v_i = \frac{m_i}{\rho_i}$$

Where m_i: mass

ρ_i density

$$v_c = \frac{m_c}{\rho_c}$$

Or $$v_c = \frac{m_c}{\rho_c} = \frac{m_1}{\rho_1} + \frac{m_2}{\rho_2} + \frac{m_3}{\rho_3} \dots\dots + \frac{m_n}{\rho_n}$$

Example: -

A cemented carbide cutting tool used for machining contains (54.7 vol.%) of WC (Tungsten carbide), (34.9 vol.%) of TiC (Titanium carbide), (4 vol.%) TaC (Tantalum carbide), and (6.4 vol.%) Co (Cobalt). Estimate the density of the composite. Where the densities are ($\rho_{WC} = 15.77$ g/cm^3 , $\rho_{TiC} = 4.94$ g/cm^3 , $\rho_{TaC} = 14.5$ g/cm^3 , $\rho_{CO} = 8.90$ g/cm^3)

Solution:

$\rho_c = \sum V_i * \rho_i = 0$ .547*15.77 + 0.349*4.94 + 0.04*14.5 + 0.064*8.9 = 11.5 g/cm^3

Upper and Lower bound

Lower bound on apparent Young's modulus

The basic for the determination of a lower bound on the apparent Young's modulus is application of the principle of minimum complementary energy.

$$E_{(L)} = \frac{E_m * E_p}{V_m * E_p + V_p * E_m}$$

Where: E_L= modulus of composite material

E_m= modulus of basic matrix

E_p= modulus of dispersed material (particles)

V_m = volume fraction of matrix

V_p = volume fraction of dispersed material

Upper bound on apparent Young's modulus:

The basic of determination of an upper bound on the apparent Young's modulus is application of the principle of minimum potential energy.

$$E_{(U)} = E_p * V_p + E_m * V_m$$

There are many considerations must be taken in the account of choosing of dispersed material (particles) which are:

1- No chemical reaction with the matrix.
2- Hard and solid to obstacle the slip (dislocation movement).
3- Stable at high temperature and insoluble in matrix.

The following formula is used to determine the activity of dispersed particles as obstacle to dislocation movement as following:

$$D_p = \frac{2*d^2}{3*v_p}(1 - v_p)$$

Where: v_p= volume fraction of particles

d = particle diameter

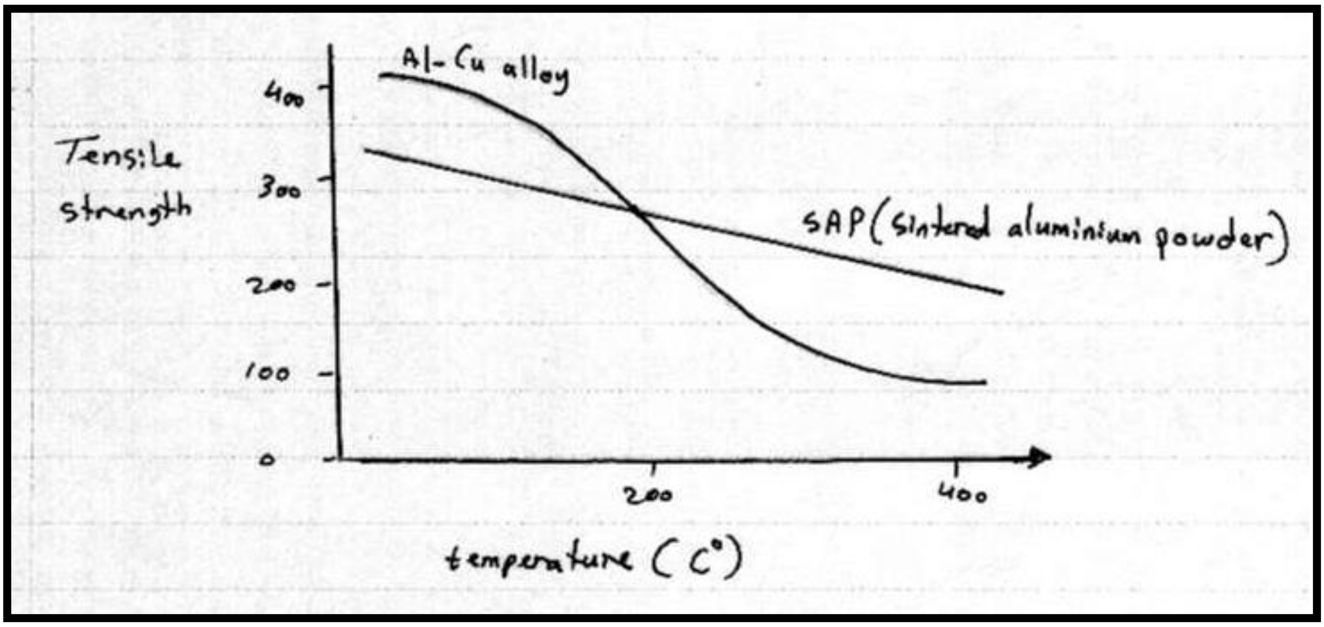
D_p= the distance between particles

Types of Hardening

there are two **types of hardening**

- **dispersion hardening**
  - insoluble second phase in a soft metalic matrix
- **precipitation hardening (age hardening)**
  - it is most widly used method of strenthening alloys.(i.e. heat treatment followed by rapid cooling) that caused precipitation

The (dispersion) sintered aluminum has advantage over precipitation hardened aluminum alloys in that it retains its strength better at high temperature as showing:



This is because at the higher temperatures, the precipitate particles in precipitation hardened alloys tend to coalesce or go into solution in the metal. While in dispersion composite material the particle remains (insoluble) to obstacle the dislocation movement.

2.2 – Natural Particulate Composite

There are some examples of naturally composite

Wood -
- Matrix (lignin)
- Reinforcement (cellulose fiber)

Bone and teeth -
- Matrix (collagen)
- Reinforcement (Calcium and phosphate ions and hydroxyapatite)

2.3 - Nano Composite:

Is a multiphase material in which at least one of the phases has at least one dimension in order of nanometers (less than 100nm, i.e. 0.1 μm)

Bone – natural nano-composite.

3- Principles of Fiber Reinforcement

3.1- Type, Classification and Properties

The fibers may be continuous throughout the matrix or short fibers, and aligned in all the same direction or randomly arranged as shown in figure (1). Like glass or carbon fibers in polymers and ceramic fiber in metal.

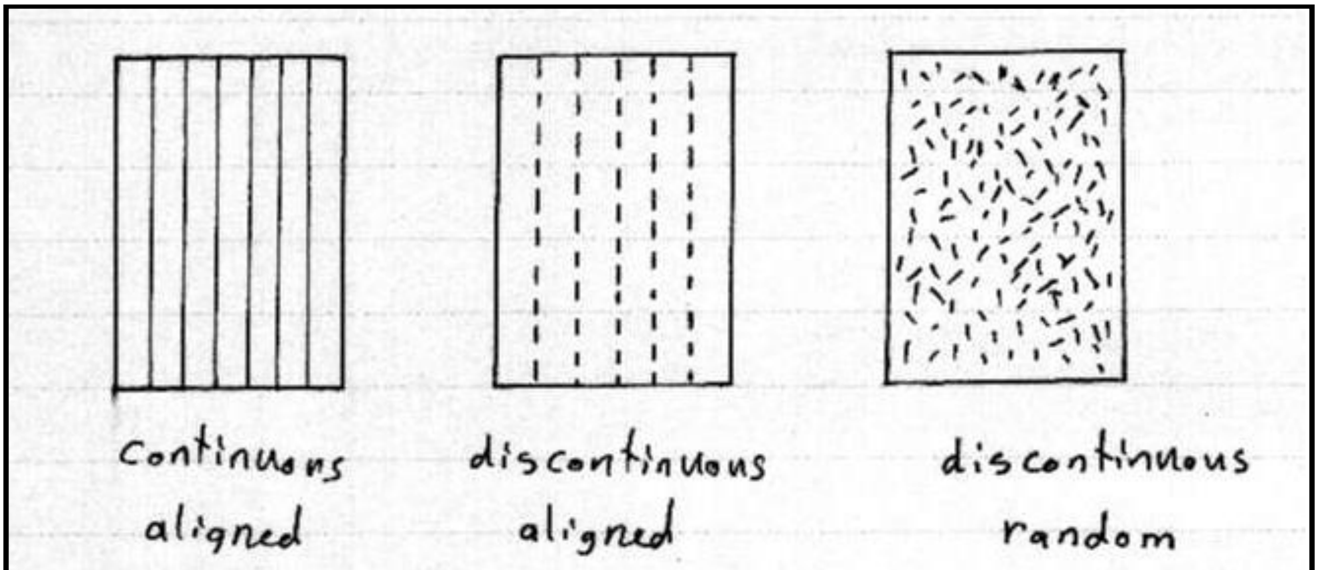


Fig. (1)

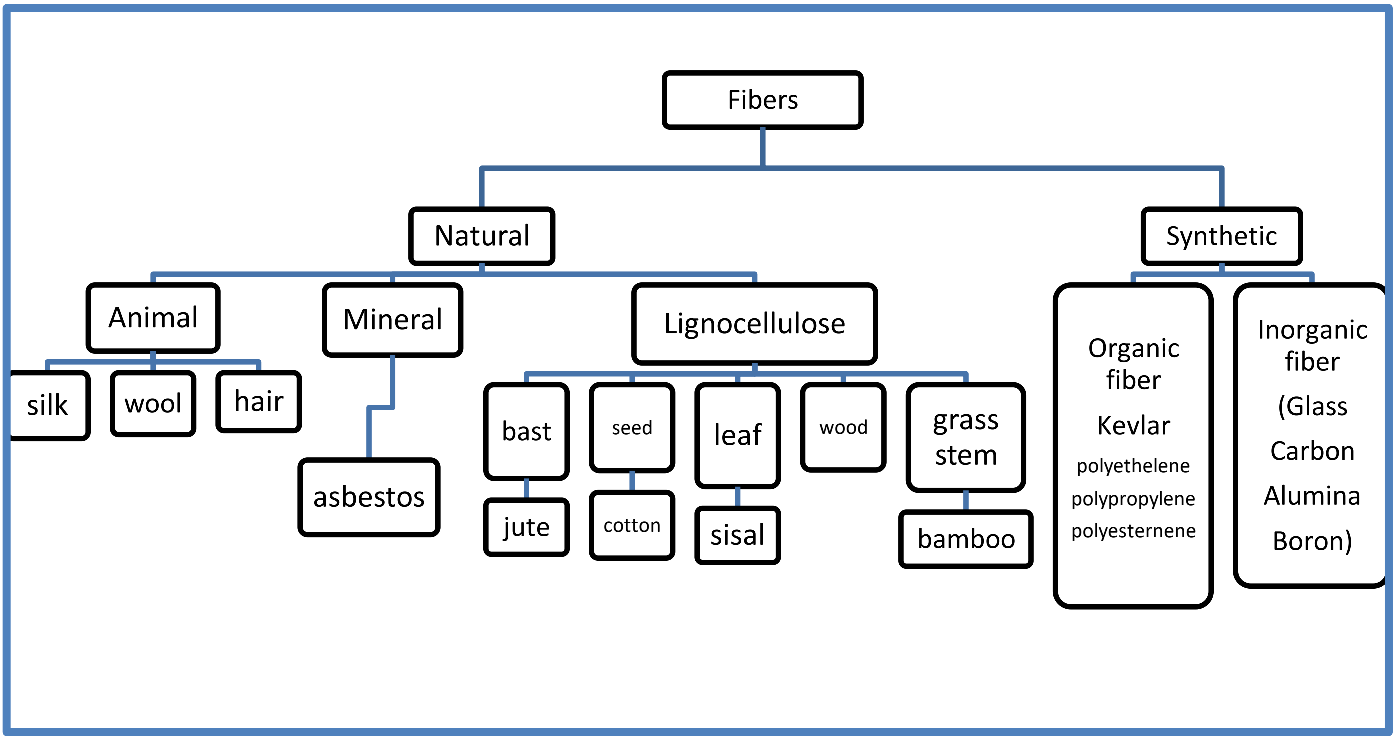
Fibrous form gives the rigidity and strength of the composite

Continuous fiber composite: A composite whose reinforcement is made by fibers of indefinite length.

Discontinuous fiber composite: A composite whose fibers have a limited length, typically (3-50) mm.

Unidirectional composite: A composite in which the fibers are aligned according to the principal orientation.

Fibers may be classified according to its nature as following:



3.2- Common fiber used in composite materials

The most common fibers used for engineering applications are:

- Glass
- Carbon (graphite)
- Kevlar (aramid)
- Boron

While the most widely used as a matrix for fibrous composites are epoxy, polyester and organic material "supper polymer" commonly called plastics.

Properties of fiber

1- High modulus of elasticity.
2- High ultimate strength.
3- Low variation of strength between individual fibers.
4- Uniform fiber cross- section.
5- Stability and retention of strength during fabrication.

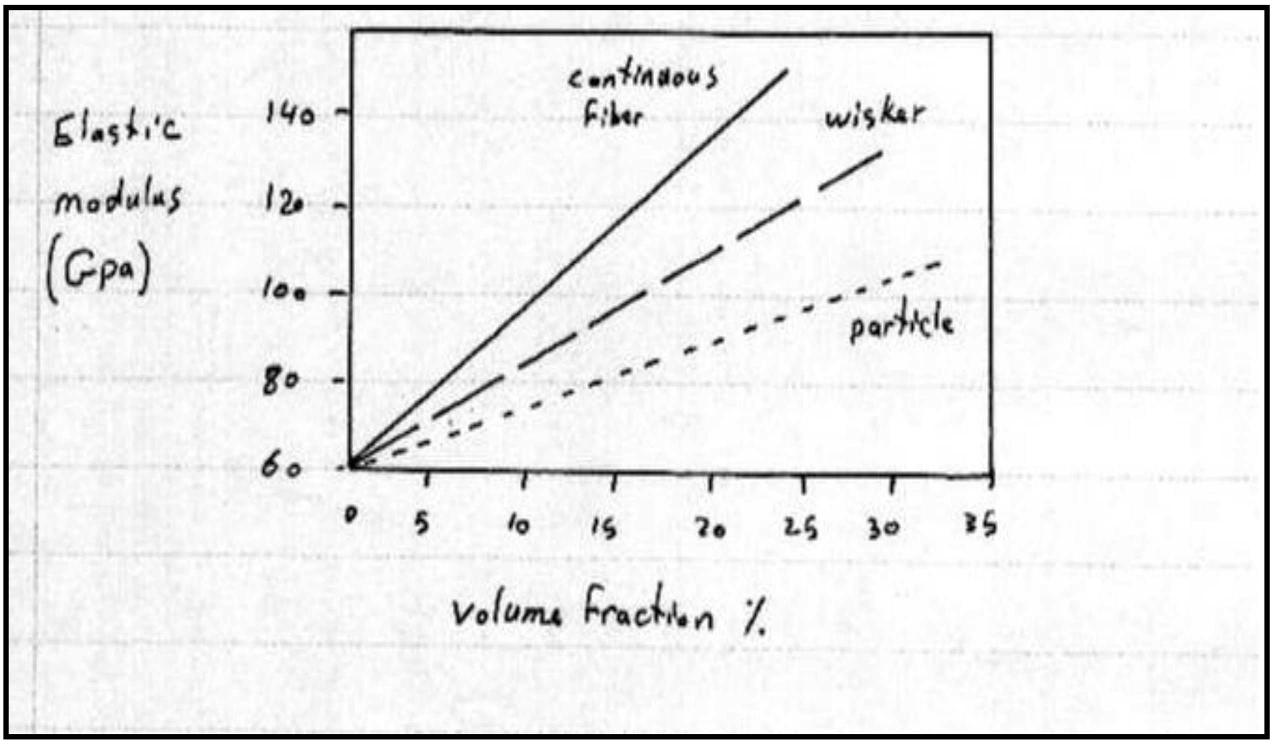
Properties of matrix

1- Low density.
2- Low strength.

3- Bind the fibers together.
4- Transfer the load to the fiber.
5- Stop, to some extent, a crack from propagation.
6- Protect the fibers surfaces from damage in service.
7- Be chemically and thermally compatible with fiber.

Fibers may be wires or whisker (**whiskers** are very fine fibers have short length to diameter ratio).

The following figure shows the increase modulus with increase the volume fraction for the same reinforcement but different shape (continuous fiber, whisker or particle). This schematic shows the loss of reinforcement efficiency as one goes from continuous fiber to particle.



The most fibers used in composite material are :-

1) **Glass fiber**

- The most common and inexpensive fiber, usually used for the reinforcement of polymer matrices.
- Typical composition is (50-60% SiO_2), and other oxides of (Al, Ca, Mg, Na,…….etc.).
- Glass fibers are available as:

a) Chopped strands.

b) Continuous yarn.

c) Roving.

d) Fabric sheet.

- Properties of Glass fiber

- Good dimensional stability.
- Resistant to heat.
- (Strength - to – density) is high.

2) **Carbon fibers**

a) Carbon is very light element, with density about (2.39 gm/cm^3).

b) Carbon has excellent compression properties.

c) Good thermal properties.

d) Carbon fiber adds electrical conductive properties to composite.

3) **Ceramic fiber**

❖ Such as { Alumina (Al_2O_3) / Silicon carbide (SiC) }

❖ It is used in very high temperature applications.

❖ It has poor properties in tension and shear.

4) Metallic fiber

- ❖ Such as steel and tungsten.
- ❖ Have high strength.
- ❖ Density is very high for these fibers.

5) Natural fibers

- ➢ Cotton
- ➢ Flax (الكتان)
- ➢ Jute
- ➢ Hemp (القنب)
- ➢ Ramie
- ➢ Wood
- ➢ Straw (القش)
- ➢ Hair
- ➢ Wool
- ➢ Silk

3.3- Rule of Mixture in Fibrous Composite

As for particulate composites, the rule of mixtures always predicts the density of fiber – reinforced composites.

The mass (m_c) of a composite is made up of masses of the matrix (m_m) and the fiber (m_f) i.e.

$$m_c = m_m + m_f \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

Since mass is volume (v) time density (ρ) then equation (1) can be written as:

$$v_c * \rho_c = v_m * \rho_m + v_f * \rho_f$$

And so:

$$\rho_c = \frac{v_m}{v_c}\rho_m + \frac{v_f}{v_c}\rho_f$$

$\frac{v_m}{v_c}$ is the matrix volume fraction (V_m)

$\frac{v_f}{v_c}$ is the fiber volume fraction (V_f)

Thus,

$$\rho_c = V_m * \rho_m + V_f * \rho_f \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

And $V_m = 1 - V_f$

Therefore, equation (2) can be termed a law of mixtures.

In addition, the rule of mixtures accurately predicts the electrical and thermal conductivity of fiber –reinforced composites along the fiber direction

$$K_c = V_m * K_m + V_f * K_f$$

$$\sigma_c = V_m * \sigma_m + V_f * \sigma_f$$

Where: K- is the thermal conductivity

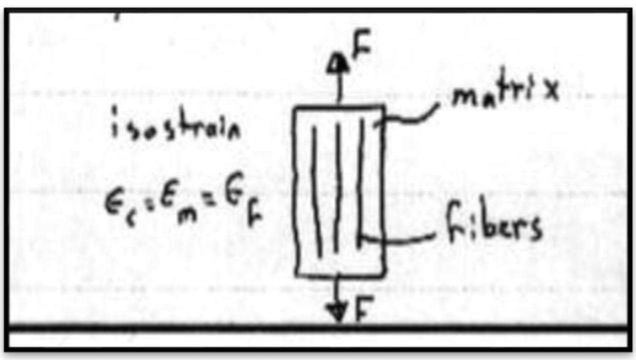
σ - is the electrical conductivity

Modulus of elasticity

The rule of mixtures is used to predict the modulus of elasticity

Parallel to the fiber (along the axis of fibers)

The total force acting on the composite is the sum of the forces carried by each constituent



$$F_c = F_m + F_f$$

Scine, $F = \sigma * A$

$$\sigma_c * A_c = \sigma_m * A_m + \sigma_f * A_f$$

$$\sigma_c = \sigma_m \left(\frac{A_m}{A_c}\right) + \sigma_f \left(\frac{A_f}{A_c}\right)$$

The area fraction (A) equal to the volume fraction (V)

$$\sigma_c = \sigma_m * V_m + \sigma_f * V_f$$

from Hooke's law, **σ =E*ε**, therefore

$$E_c * \varepsilon_c = E_m * \varepsilon_m * V_m + E_f * \varepsilon_f * V_f$$

If the fibers are rigidly bonded to the matrix, both the fibers and matrix must stretch equal a mounts (iso-strain conditions)

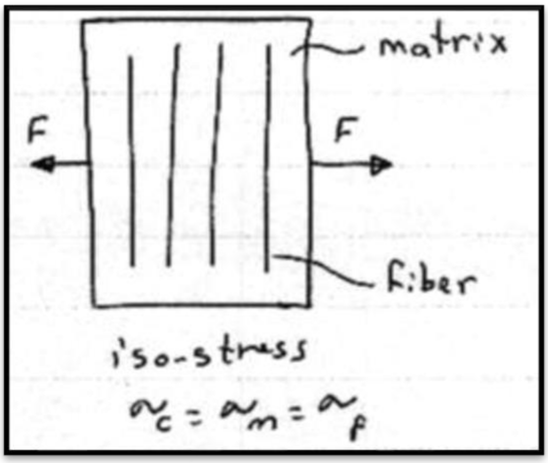
$$\varepsilon_c = \varepsilon_m = \varepsilon_f$$

so, $E_{c,II} = E_m * V_m + E_f * V_f$ *(upper bound)*

The modulus of elasticity may be high.

<u>While in perpendicular direction</u> (at right angle to fiber)

The sum of strains in each component equals to the total strain in the composite, whereas the stresses in each component are equal (iso- stress condition).



$$\varepsilon_c = \varepsilon_m * V_m + \varepsilon_f * V_f$$

$$\frac{\sigma_c}{E_c} = \frac{\sigma_m}{E_m} * V_m + \frac{\sigma_f}{E_f} * V_f$$

Science $\sigma_c = \sigma_m = \sigma_f$

$$\therefore \frac{1}{E_{c\perp}} = \frac{V_m}{E_m} + \frac{V_f}{E_f} \Longrightarrow$$

$$E_{c\perp} = \frac{E_m * E_f}{E_m * V_f + E_f V_m} \quad (lower\ bound)$$

It can also be shown, for longitudinal loading that the ratio of load carried by the fibers to that carried by the matrix is:

$$\frac{F_f}{F_m} = \frac{E_f * V_f}{E_m * V_m}$$

This can be proved as following:

In the longitudinal direction both matrix and fiber have equal strain if bonding is good, so:

$$\varepsilon_c = \varepsilon_m = \varepsilon_f$$

$$\varepsilon_m = \frac{\sigma_m}{E_m} \quad and \quad \varepsilon_f = \frac{\sigma_f}{E_f}$$

$$\therefore \frac{\sigma_m}{E_m} = \frac{\sigma_f}{E_f} \qquad \sigma = \frac{F}{A}$$

$$\frac{F_m}{E_m * A_m} = \frac{F_f}{E_f * A_f}$$

$$\therefore \frac{F_f}{F_m} = \frac{E_f * A_f}{E_m * A_m}$$

Dividing the right side by A_c

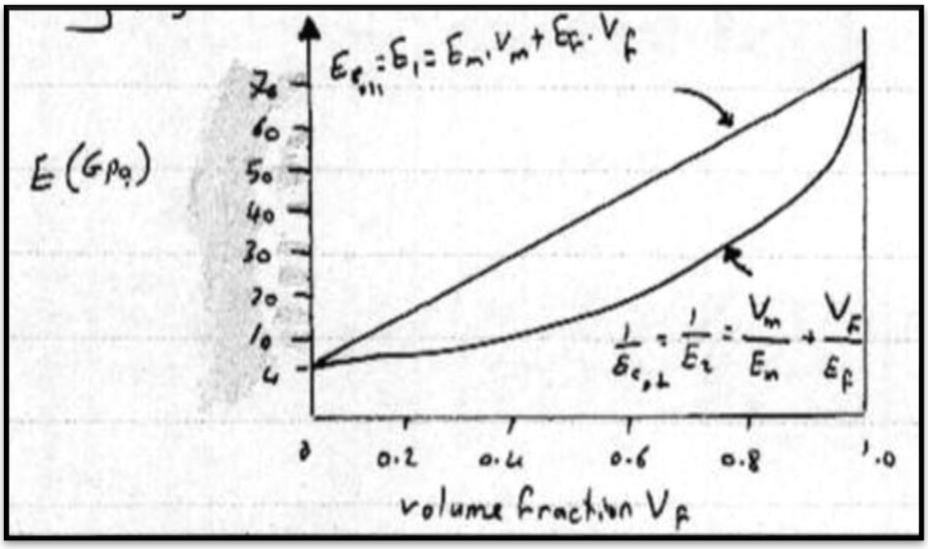
$$\frac{F_f}{F_m} = \frac{E_f * \frac{A_f}{A_c}}{E_m * \frac{A_m}{A_c}}$$

$$V_f = \frac{A_f}{A_c} \quad and \quad V_m = \frac{A_m}{A_c} \quad for\ the\ same\ length\ (L)$$

$$\therefore \frac{F_f}{F_m} = \frac{E_f * V_f}{E_m * V_m}$$

It is representing the ratio of the load carried by the fibers to that carried by the matrix.

According to the formula, the volume fraction has effect on the modulus of elasticity in both directions. For example, Rule- of –mixtures Prediction for longitudinal (E_1) and transverse (E_2) modulus for glass- polyester composite (E_f = 73.7 GPa), (Em = 4 GPa) as shown in the following figures.



Note: To calculate the volume fraction of fiber and matrix

3.4 – Relationship between Volume Fraction and Weight Fraction

1. **In term of weight fraction**

$$V_f = \frac{\rho_c}{\rho_f} * w_f \qquad , \qquad V_m = \frac{\rho_c}{\rho_m} * W_m$$

(w_f) is the **weight fraction of fiber =** $\frac{W_f}{W_c}$

(W_f) means **fiber weight ,** (W_c) means **composite weight**

(w_m) is the **weight fraction of matrix =** $\frac{W_m}{W_c}$

(W_m) **means matrix weight**

2. **In term of volume fraction**

$$V_f + V_m = 1$$

$$V_f = \frac{v_f}{v_c}$$

$$V_m = \frac{v_m}{v_c}$$

Example: The borsic (boron coated with SiC) reinforced aluminum 40 % volume fibers is an important high-temperature. Estimate the density, modulus of elasticity, and tensile strength parallel to the fiber axis. Also estimate the modulus of elasticity perpendicular to the fibers.

Material	Density g/cm^2	Modulus of elasticity (GPa.)	Tensile strength MPa.
Fibers (Borosic)	2.36	380	2760
Matrix(aluminum)	2.7	69	34.5

Solution: $V_f = 0.4$ ⟹ $V_m = 0.6$

From the rule of mixtures

$$\rho_c = \rho_m * V_m + \rho_f * V_f = 2.7*0.6+2.36*0.4 = 2.56 \text{ g/cm}^3$$

$$E_{c,II} = E_m*V_m+E_f*V_f = 69*0.6+380*0.4 = 193.4 \text{ GPa}$$

$$T_{Sc} = T_{Sm}*V_m+T_{Sf}*V_f$$

$$= 34.5*0.6+2760*0.4 = 1124.7 \text{ MPa}$$

Perpendicular to the fibers

$$\frac{1}{E_{c\perp}} = \frac{V_m}{E_m} + \frac{V_f}{E_f} = \frac{0.6}{69} + \frac{0.4}{380} = 9.75 * 10^{-3}$$

$$\therefore E_{c\perp} = 102.56 \, GPa$$

Example: A continuous and aligned glass fiber – reinforced composite consist of (40%) volume fraction of glass fibers having a modulus of elasticity of (69 GPa.), and (60%) volume fraction of polyester resin, when hardened, displays a modulus of (3.4 GPa.).

a) Compute the modulus of elasticity of this composite in the longitudinal direction.
b) If the cross- sectional area is (250 mm^2) and a stress of (50 MPa). Is applied in this longitudinal direction, compute the magnitude of the load carried by each of the fiber and matrix phases.
c) Determine the strain that is sustained by each phase when the stress in part (b) is applied.

d) Compute the modulus of elasticity of this composite in the perpendicular direction.

Solution:

a) $E_{c,II} = E_m*V_m+E_f*V_f =$ 3.4*0.6+69*0.4 = 30 GPa.

b) $\frac{F_f}{F_m} = \frac{E_f*V_f}{E_m*V_m} = \frac{69*0.4}{3.4*0.6} = 13.5$

or F_f =13.5 F_m

$F_c=A_c*\sigma$ =250*50= 12500 N

This total load is just the sum of load carried by fiber and matrix

$F_c = F_f+F_m$

13.5 F_m+ F_m = 12500

F_m = 860 N

Where, $F_f = F_c - F_m$= 12500 - 860 = 11640 N

The fibers support the majority of the load.

c) $A_m=V_m*A_c$= 0.6*250 = 150 mm^2

$A_f=V_f*A_c$ = 0.4*250 = 100 mm^2

$$\sigma_m = \frac{F_m}{A_m} = \frac{860}{150} = 5.73\ MPa$$

$$\sigma_f = \frac{F_f}{A_f} = \frac{11640}{100} = 116.4\ MPa$$

Finally, the strain is computed as:

$$\varepsilon_m = \frac{\sigma_m}{E_m} = \frac{5.73}{3.4*10^3} = 1.69*10^{-3}$$

And , $\varepsilon_f = \frac{\sigma_f}{E_f} = \frac{116.4}{69*10^3} = 1.69*10^{-3}$

$$\varepsilon_c = \frac{\sigma_c}{E_c} = \frac{50}{30 * 10^3} = 1.69 * 10^{-3}$$

d) $E_{c\perp} = \frac{E_m * E_f}{V_m * E_f + V_f * E_m} = \frac{3.4*69}{0.6*69+0.4*3.4}$

$E_{c\perp}$ = 5.5 GPa.

Example: For a glass fiber- epoxy matrix composite with the volume fraction of fiber as (65%). Estimate the modulus of elasticity when the load is at (0^o) with the fibers and the modulus of elasticity when the load is at (90^o) with the fiber.

Note: modulus of elasticity for epoxy = 3.5 GPa.

modulus of elasticity for glass fiber = 70 GPa.

Solution:

$E_{c,II} = E_m * V_m + E_f * V_f$ = 3.5*0.35+70*0.65 = 47 GPa

$$\frac{1}{E_{c\perp}} = \frac{V_m}{E_m} + \frac{V_f}{E_f} = \frac{0.35}{3.5} + \frac{0.65}{70} = 0.11 \rightarrow E_{c\perp} = 9 \ GPa$$

Example: Consider a uniaxial fiber reinforced composite of aramid fibers in an epoxy matrix. The volume fraction of fibers is (60 %). The composite is subjected to an axial strain of (0.1 %). Compute the modulus and strength along the axial direction of the composite, E_f=140 GPa (aramid fiber), E_m=5 GPa (epoxy)

Solution:

$E_{c,II} = E_f * V_f + E_m * (1 - V_f)$

$E_{c,II}$ = 140*0.6+5*0.4 = 86 GPa

$$\sigma_{cII} = \varepsilon * E$$

$$\sigma_{cII} = 0.001 * 86 = 86 \ MPa$$

Example: What is the ratio of the longitudinal modulus of elasticity to the transverse modulus for a composite with continuous aligned fiber constituting (50 %) of the volume if the tensile modulus of the fiber (50 times) that of the matrix.

Solution:

$E_{c,II} = E_m*V_m+E_f*V_f$

$= E_m*0.5+50\ E_m*0.5 = 25.5\ E_m$

a) $E_{c\perp} = \frac{E_f*E_m}{E_m*V_f+E_f*V_m}$

$$= \frac{50\ Em*\ Em}{Em*0.5+50\ Em*0.5} = \frac{50\ Em}{25.5} = 1.961Em$$

$$\therefore \frac{EcII}{E_{c\perp}} = \frac{25.5\ Em}{1.961Em} = 13.005$$

Example: A composite material has a longitudinal modulus of elasticity of (18.2 GPa). Containing unidirectional S – glass fibers in on epoxy matrix. Determine.

a) Volume fraction of glass fiber and the epoxy matrix.
b) The density of the composite.
c) The ratio of load carried by the fibers to that carried by the matrix.

Note: Density of epoxy = 1.3 gm/cm^3

Density of glass = 2.2 gm/cm^3

Modulus of epoxy = 2.75 GPa.

Modulus of glass =380 GPa.

Solution:

a) $E_c = E_m*(1-V_f) + E_f*V_f$

$18.2 = 2.75*(1- V_f) + 380* V_f$

$V_f = 0.041$ ⟹ $V_m= (1- V_f) = 0.959$

b) $\rho_c = \rho_m * V_m + \rho_f * V_f$

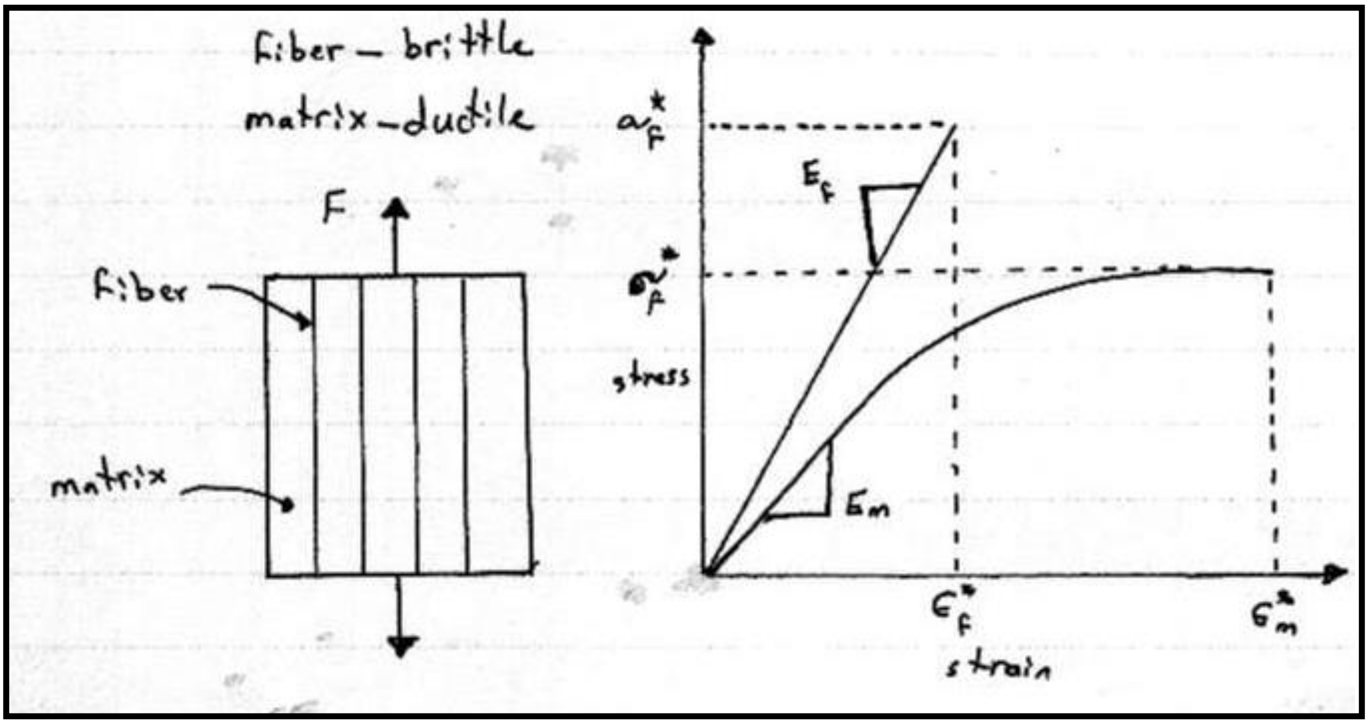
$=1.3*0.959+2.2*0.041$

$=1.2467+0.9902 = 1.3369$ gm/cm^3

c) $\frac{F_f}{F_m} = \frac{E_f*V_f}{E_m*V_m} = \frac{380*0.041}{2.75*0.959}$ =5.91

4.1 - Stress-Strain Behavior of Aligned Fiber Composite

The following figure represented schematically stress-strain behaviors for the fiber and matrix (loaded in the longitudinal direction).



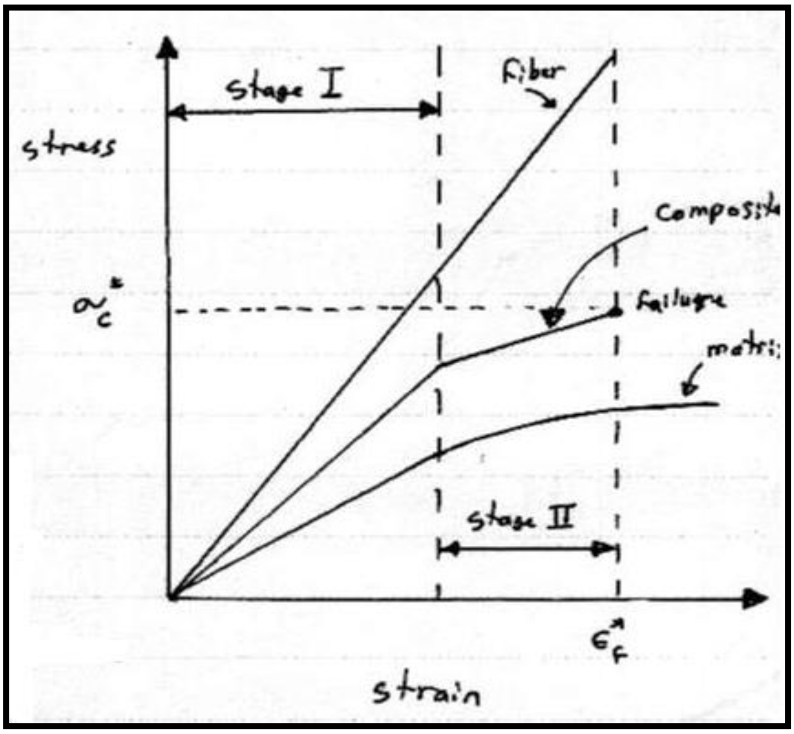
- In the stage I region,

  Both fibers and matrix deform elastically.

- In stage Π

The matrix starts to yield as the fibers continue to deform elasticity.

-The onset of composite failure begins as fibers start to fracture.



4.2 – Causes of Composite Failure

The main causes of failure in fibrous composites may be occur:

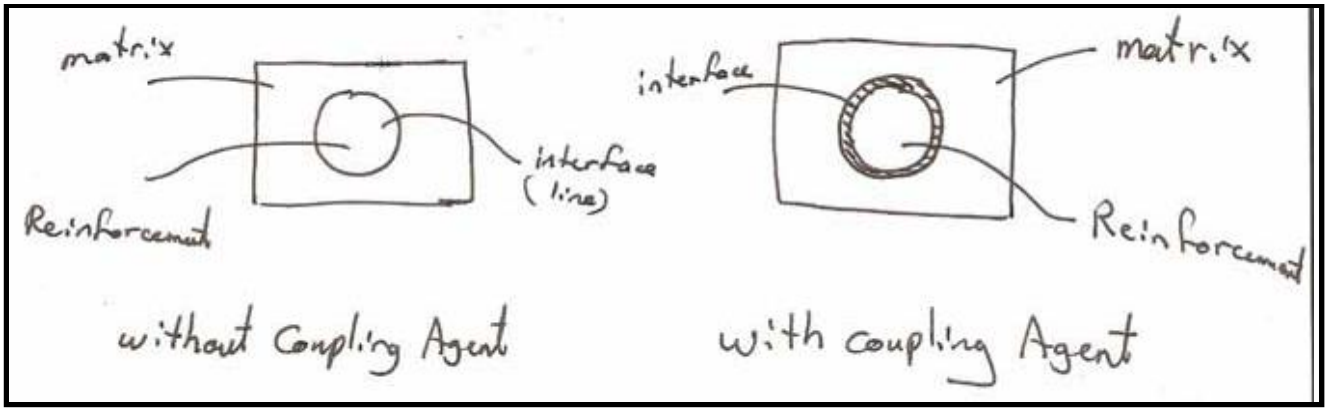
a) Breaking of fibers.
b) Debonding of fibers from matrix (separation of fibers and matrix).
c) Micro cracking of the matrix.
d) Delamination.

4.3 - Components of Composite Materials

1. Matrix
2. Reinforcement
3. Interface

5.1 – Definition of Interface

The interface is a bonding surface or zone between the reinforcement and matrix.



- The matrix material must be "wet" the reinforcement materials. Coupling agents are frequently used to improve wettability. Also, well wetted reinforcement leads to increase the interface surface area and increase bonding.

- Bonding with the matrix can be either weak vander-walls forces or strong covalent bonds.

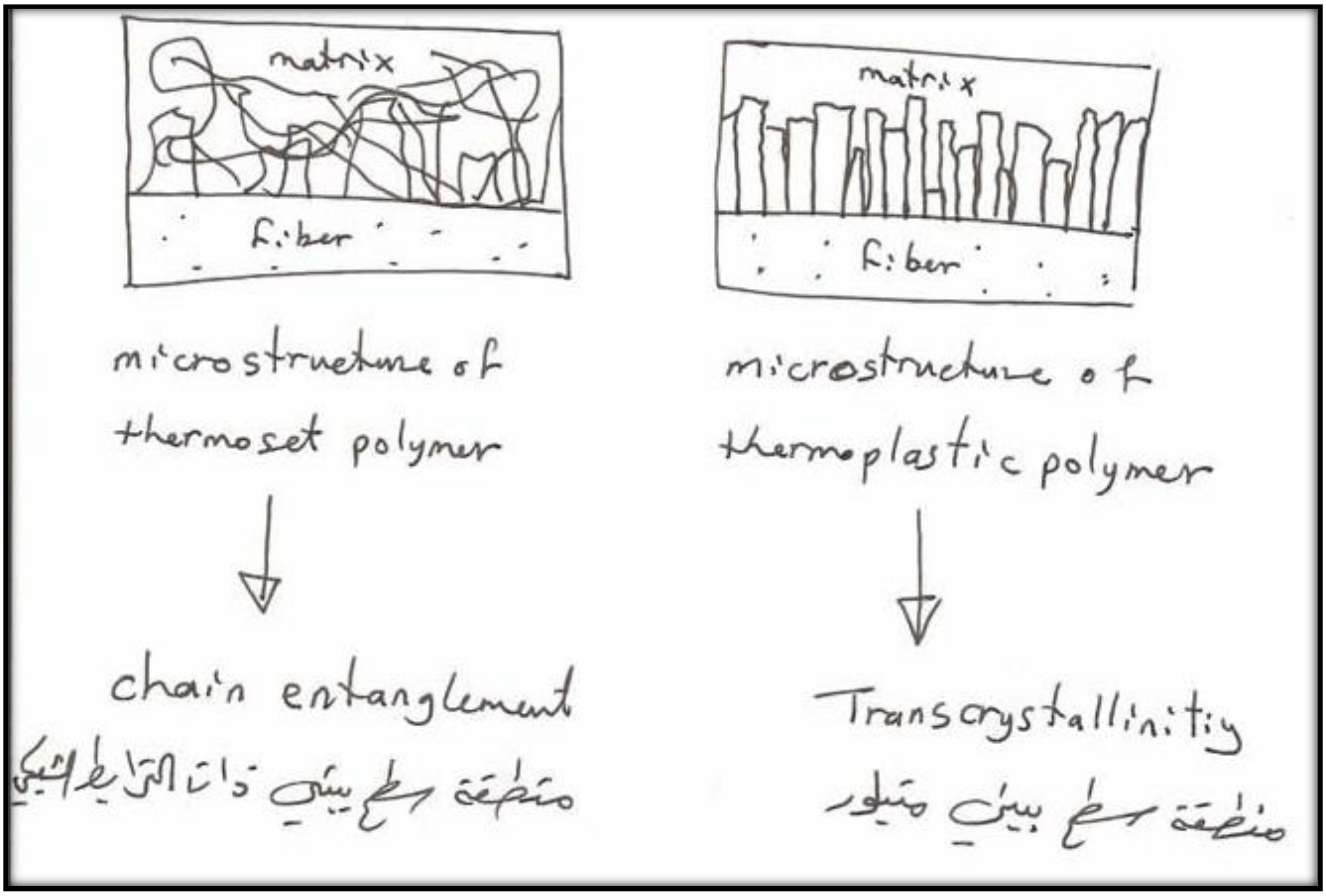
- To obtain desirable properties in a composite, the applied load should be effectively transferred from the matrix to the fibers via the interface. This means that the interface must be larger and exhibit strong adhesion between the reinforcement and matrix.

- The internal surface area of the interface can go as high as 3000 cm^2/cm^3.

- The coupling agents form the interphase which has different mechanical properties from that of matrix and reinforcement. These interphase properties play a very important role in determining the ultimate properties of the composite materials.

- The interphase is transfer the mechanical stress between the matrix and the reinforcement. Therefore, the mechanical properties depend on the properties of the interphase also.

- An interface is a two dimensional construction an area having a common boundary between the constituents. Whereas an interphase is a three dimensional phase between the constituents and has its high properties.



5.2 – General Requirement of the interphase

- Big bond
- chemical stability

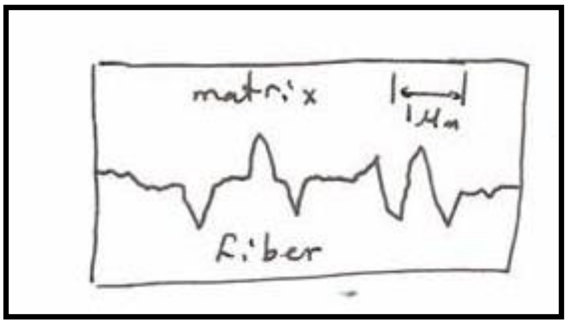
} In order to carry the load from the matrix to reinforcement

Therefore, the interphase depends on

1) Reinforcement shape.
2) Surface roughness of the reinforcement.
3) Treated the surface by coupling agent (wettability).

5.3 – Type of Bond

5.3.1 - Mechanical Bond



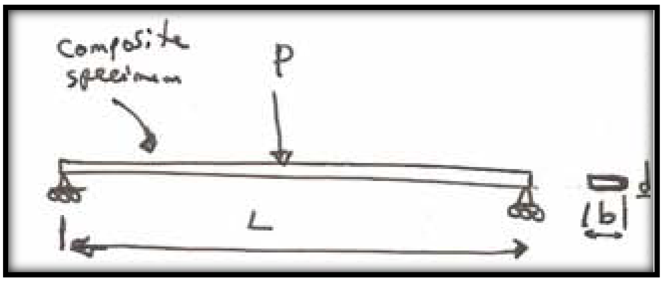
That depends on surface roughness.

5.3.2 - Chemical Bond

The chemical bond occurs by covalent, ionic and metallic bonds, when the wettability increase → increase chemical bonding.

- Failure at the interface (called deboning)
- The interfacial strength (max. shear stress)

Interfacial strength is measured by simple tests that induce adhesive failure between the reinforcement fibers and the matrix. The most common test is the three – point bend test or interlaminar shear stress test by founding max. Shear stress (τ_{max}).



$$\tau_{max} = \frac{3P}{4bd} \qquad (MPa)$$

Where:

P= force at the fracture (N).

b = width of the composite specimen (mm).

d = thickness of the composite specimen (mm).

5.4 - Advantages of Composite Materials

1) High resistance to fatigue and corrosion degradation.
2) High strength - to - weight ratio, as shown in the following table.

Material	Strength (Ib/in^2)	Density (Ib/in^3)	Strength-to-weight ratio (in)
Polyethylene	1000	0.030	$0.03*10^6$
Pure aluminum	6500	0.098	$0.07*10^6$
Epoxy	1500	0.050	$0.3 *10^6$
Alloy steel	240000	0.28	$0.86 *10^6$
Aluminum alloy	86000	0.098	$0.88*10^6$
Titanium alloy	170000	0.16	$1.06 *10^6$
Carbon-carbon composite	60000	0.065	$0.92 *10^6$
Carbon-epoxy composite	80000	0.050	$1.6*10^6$

3) Due to greater reliability, there are fewer structural repair.
4) Composite are dimensionally stable, i.e. they have low thermal conductivity and low coefficient of thermal expansion.
5) Manufacture and assembly are simplified.

5.5 - Disadvantages of Polymers in Construction are

1) High cost of materials.
2) Low stiffness and strength.
3) Poor scratch resistance.
4) Degradation under UV light (stabilizers used)
5) Low resistance to fire and high temperature (additive used).

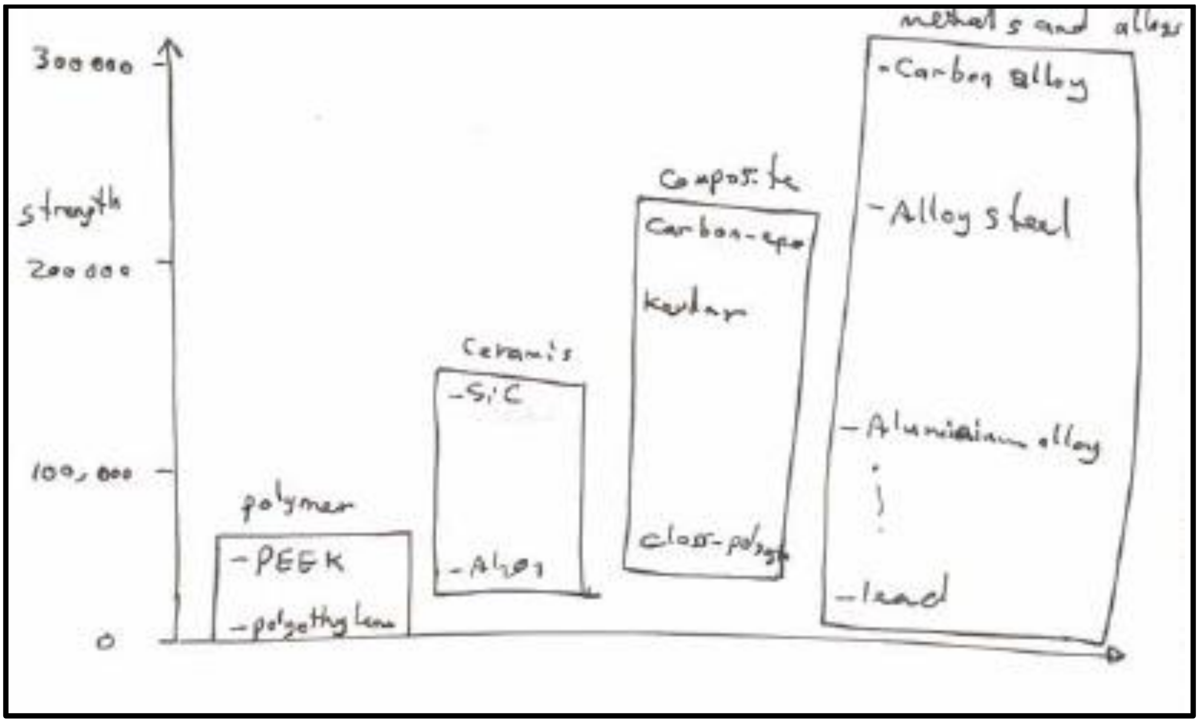
6.1 - Limitation of Composite Materials

1) High cost of raw materials and fabrication.
2) Composites are more brittle than metals and thus are more easily damaged.
3) Transverse properties may be weak.
4) Reuse may be difficult.

5) Difficult to attach.
6) Analysis is difficult.

Comparison between Composite and Metals

1) Composites offer significant weight saving over existing metals. Composite can provide structures that are (25-45%) lighter than the conventional metal structure for the save functional requirement. This is due to lower density of the composites. i.e. densities of composites range from (1.26-1.82 gm/cm^3) as compared to (2.8 gm/cm^3) for aluminum.
2) Unidirectional fiber composite has specific tensile strength (ratio of material strength to density) about (4- 6) times greater than that of steel and aluminum.
3) Unidirectional composites have specific – modulus (ratio of the material stiffness to density) about (3 -5) times greater than of steel and aluminum.



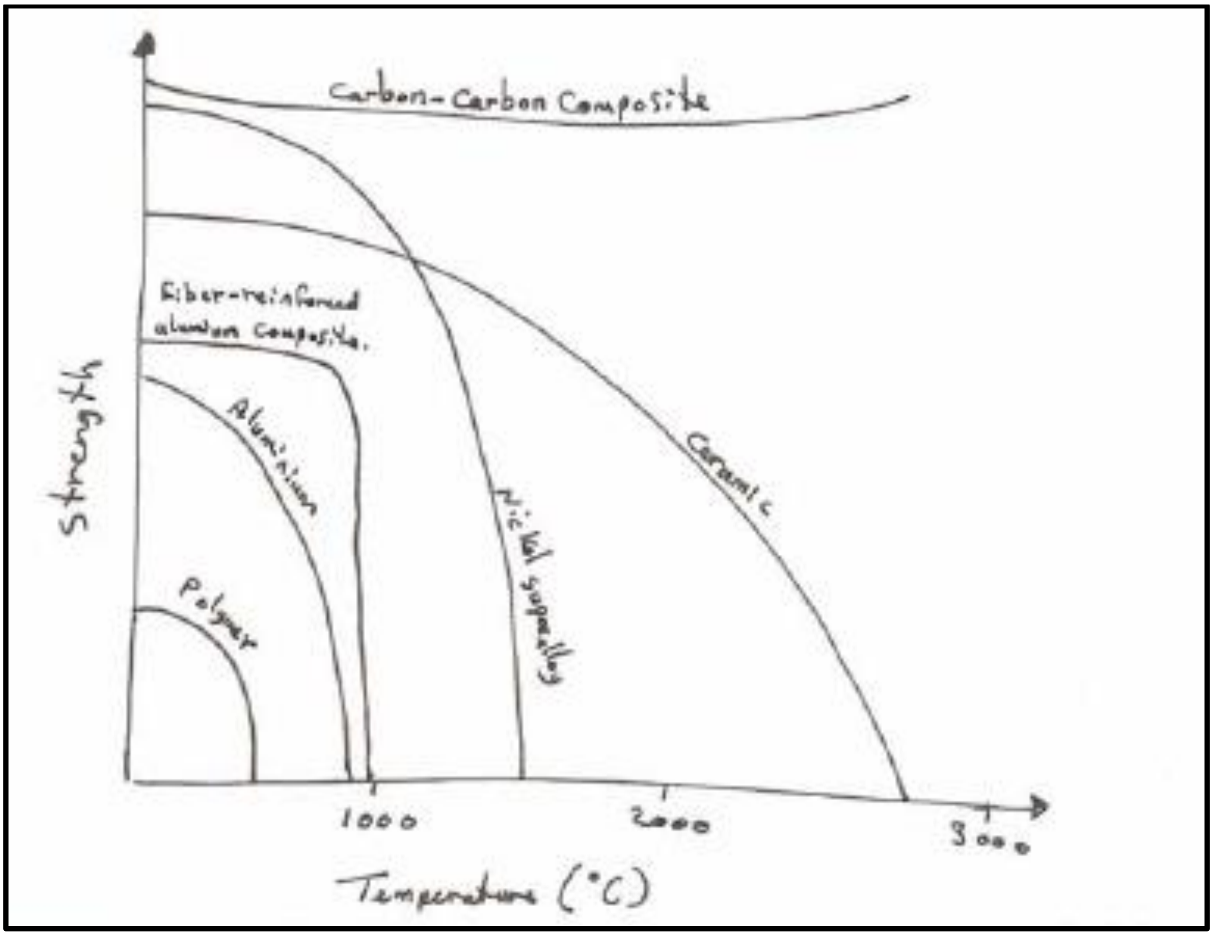
6.2 - Durability of Polymer Composites

Polymer composites change with time and most significant factors are:

1) Elevated temperature.
2) Fire.
3) Moisture.
4) Adverse chemical environments.
5) Natural weathering when exposed to sun’s ultra- violet radiation.

6.2.1 - Temperature

- ❖ Fluctuating temperature have greater deterioration effect on the properties of composites. Different in coefficient of thermal expansion coefficient of reinforcement and matrix may cause debonding.
- ❖ Exposed to high temperatures lead to discoloration of the resin becoming yellow. As a result of exposure to high temperature, the composite becomes brittle.
- ❖ The effect of temperature on strength of materials represented by the following figure.



6.2.2 - Firing

A composite material must meet appropriate standards of fire performance.

- Aluminum trihydrate
- Antimony trioxide

} Are used as fillers to enable flame – retardant properties.

6.2.3 - Moisture

Polymer absorbs water which may cause a decrease in strength and modulus of elasticity. Absorption of water by polyesters and epoxies lead to swelling of laminate.

Water will also cause some surface flaws on fibers, long-term of water absorption may cause weakening of the bond between fiber and polymer.

6.2.4 - Chemical Environments

6.2.5 - Weathering

Natural weathering can affect mechanical properties of composite through surface deboning.

Because of weathering is surface effect, thickness of laminate becomes important.

3mm thickness ⟹ (12-20%) reduction in flexural stress after 15 years

10 mm thickness ⟹ ~ 3% reduction in flexural stress after 50 years

7- Mechanics and Principles of Fiber Reinforcement

Many factors must be considered when designing a fiber – reinforced composite, including the length, diameter, orientation, amount, properties of the fibers, properties of matrix and the bonding between fibers and matrix.

7.1 - Influence of Fiber Length

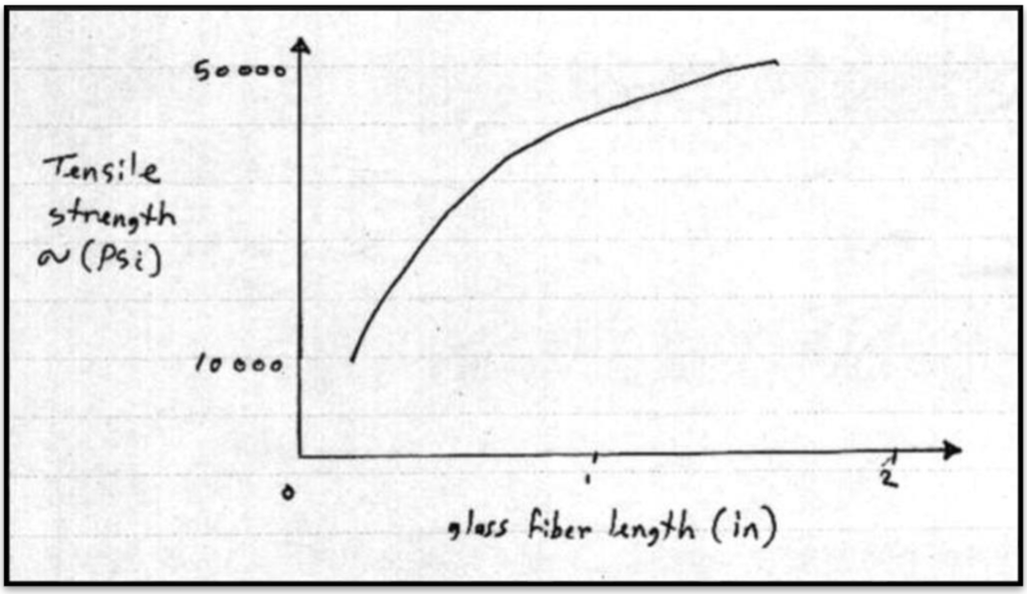
Length of short discontinuous fibers has greater effect on the properties of the composite materials.

Fibers dimensions are often characterized by the aspect ratio $(\frac{l}{d})$ where

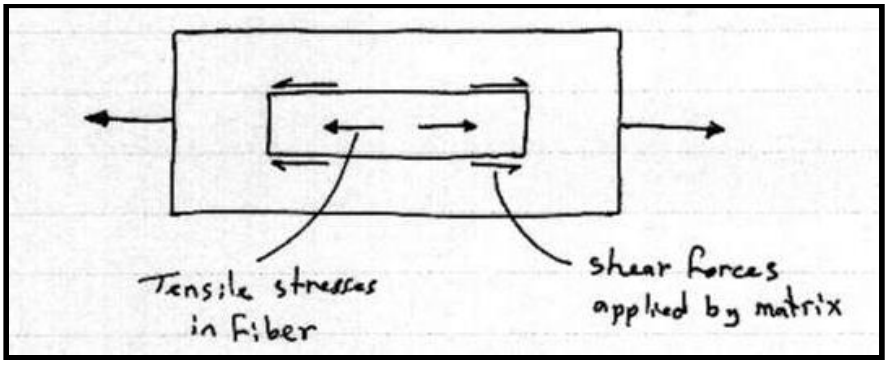
l - Fiber length

d - Fiber diameter

The strength of composite improves when the aspect ratio is large. For example; increasing the length of chopped E – glass fibers in an epoxy matrix increases the strength of the composite. As shown in figure below:



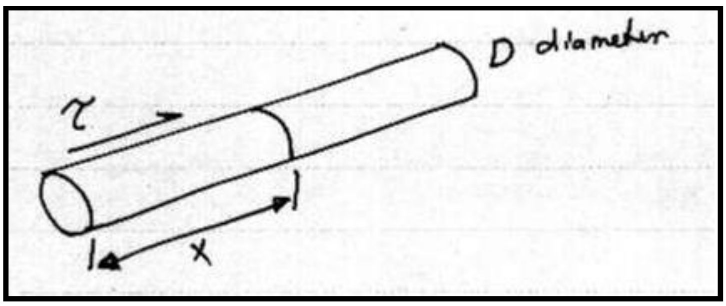
When a load is applied to a composite it is applied to the matrix and transferred to the fibers by some combination of shear and tensile stresses acting across the interface. As shown in following figure.



The discontinuous fiber in the matrix stretched as a result of interfacial shear stresses acting on the surfaces of the fiber.

Theses shear stresses will be maximum at the ends of the fiber. While the tensile stresses are zero at the ends and maximum at the middle.

Consider the interfacial shear stress acting on single fiber in a matrix in the following figure.



If (τ) is the average interfacial shear stress, then the shear force acting on a section of the fiber length (x) and of uniform cross-sectional diameter (D) is the:

Shear force = shear stress * area

$$= \tau * \pi D x$$

This shear force is equal to normal force on the fiber, therefore

$$\sigma_f * \frac{1}{4}\pi D^2 = \tau * \pi D x$$

and so $$\sigma_f = \frac{4\tau x}{D}$$

The stress increases from zero at the end of a fiber, i.e. When (x=0) , to its maximum possible value when ($X = \frac{1}{2}$ Lc)

Hence, the maximum value of the tensile stresses is given by:

$$\text{Maximum } \sigma_f = \frac{2 * \tau * Lc}{D}$$

Then the critical fiber length (Lc), for any given fiber diameter (D) can be determined

$$Lc = \frac{\sigma_f * D}{2 * \tau}$$

Critical length: It is the minimum length at which the tensile stress in the fiber reaches the maximum value.

If the fiber length ($L=L_C$)

The stress position profile shown in fig. (1, a). It can be seen that the maximum fiber load is achieved only at the axial center of the fiber.

If the fiber length ($L > L_C$)

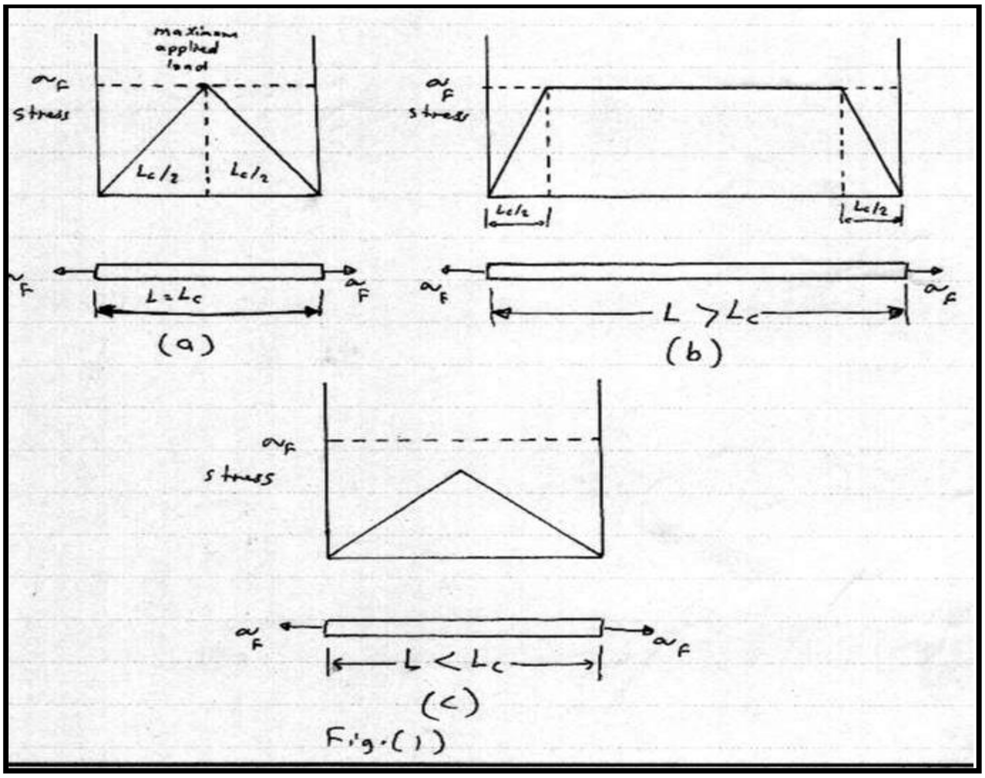
The fiber reinforcement becomes more effective as shown in fig. (1, b).

If the fiber length ($L < L_C$)

It is observing little reinforcing effect as shown in fig. (1, c).

When fibers have length($L>>L_C$) (normally $L>>15L_C$)

Are termed continuous.



Summary

For $L<L_C$ strength is relatively low (low effective)

For $L=L_C$ moderate strength

For $L>L_C$ strength is relatively high (more effective)

For $L>>L_C$ ($L>15L_C$) strength equivalent to iso- strain model (Continuous)

Calculations of Average Stress

Average stress $= \left[1-\frac{Lc}{2L}\right]\frac{2\tau Lc}{D}$ for $L > Lc$

$= \left[1-\frac{Lc}{2L}\right] * \sigma_{f\,max}$

Average stress $=\frac{\tau Lc}{D}$ for $L = Lc$

Average stress $=\frac{\tau L}{D}$ for $L < Lc$

Calculation of the Strength of the Composite

$$\sigma_C = V_f * \sigma_f * \left[1 - \frac{L_C}{2L}\right] + V_m * \sigma_m \quad \text{when} \quad L \geq L_C$$

$$\sigma_C = V_f * \sigma_f * \left[\frac{L}{2L_C}\right] + V_m * \sigma_m \quad \text{when} \quad L \leq L_C$$

$$\sigma_C = \frac{V_f * \sigma_f}{2} + V_m * \sigma_m \quad \text{when} \quad L = L_C$$

Example: A glass fiber polyester composite contains (60%) by volume of fibers. The fibers being of length (3 mm) with diameter (0.005 mm). If the failure stress for the fibers is (1500 MPa), the shear strength (25 MPa), and the matrix has a tensile strength of (50 MPa). Determine:

- Critical length of the fiber.
- Max. average stress.
- Strength of the composite

And compare the strength when used a continuous fiber, and when used fiber equal to critical length.

Solution:

$$Lc = \frac{\sigma_f * D}{2\tau} = \frac{1500 * 0.005}{2 * 25} = 0.15mm$$

Max. average stress $= \left[1 - \frac{L_C}{2L}\right] * \sigma_{f\,max} = \left[1 - \frac{0.15}{6}\right] * 1500 = 1462.5 \text{ MPa}$

Hence, the strength of the composite is:

$$\sigma_C = V_f * \sigma_f * \left[1 - \frac{L_C}{2L}\right] + V_m * \sigma_m$$

$$= 0.6 * 1500 \left[1 - \frac{0.15}{2*3}\right] + 0.4 * 50 = 897.5 \text{ MPa}$$

When used a continuous fiber

$$\sigma_c = V_f * \sigma_f + V_m * \sigma_m = 0.6 * 1500 + 0.4 * 50 = 920 \text{ MPa}$$

When used a critical length

$$\sigma_C = \frac{V_f * \sigma_f}{2} + V_m * \sigma_m = \frac{0.6 * 1500}{2} + 0.4 * 50 = 470 \text{ MPa}$$

Example: The longitudinal modulus of elasticity for an aligned discontinuous fiber composite if the fibers constitute (40%) of the volume fraction is equal to (131GPa) , the fibers have a modulus of elasticity of (400GPa),and the matrix modulus of (5GPa) . Calculate the critical length, if the length of the fiber = (2 mm) and the critical length is less than the fiber length.

Solution:

$$E_c = E_m * V_m + \left[1 - \frac{L_c}{2L}\right] E_f * V_f$$

$$131 = 5 * 0.6 + \left[1 - \frac{L_c}{2 * 2}\right] * 400 * 0.4$$

$$\therefore L_c = 0.8 \text{ mm}$$

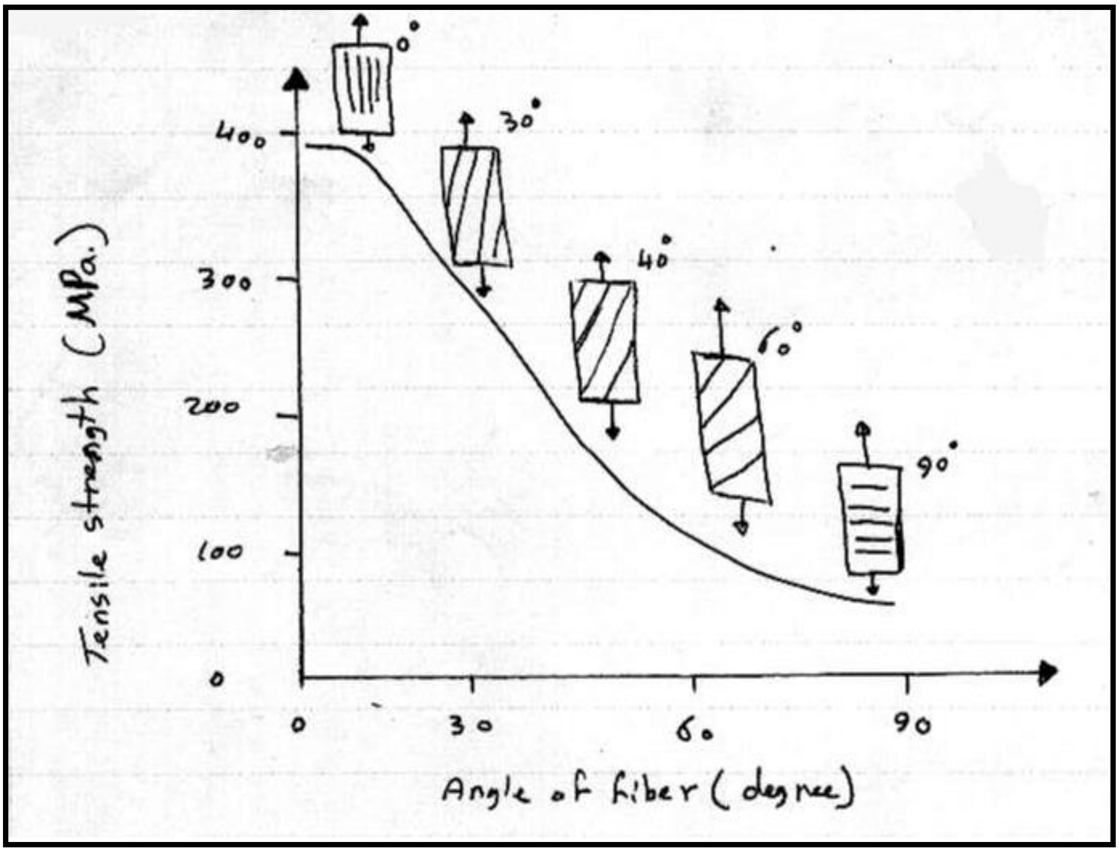
7.2 - Influence of volume fraction of fibers

A greater volume fraction of fibers increases the strength and stiffness of the composite, as expected from the rule of mixtures. Howerer, the maximum volume fraction is about (80%) beyond fibers can no longer be completely surround be the matrix.

7.3- Influence of fiber orientation and concentration

The arrangement or orientation of fibers, fiber concentration and the distribution all have a significant influence on the strength and other properties of fiber –reinforced composite.

Unidirectional arrangements of fibers properties give good strength these happen at (0°). From the following figure, it can be seen the effect of fiber angle on the tensile strength. However, unidirectional orientations provide poor properties if the load is perpendicular to the fiber.



8- <u>Structural Composite (Laminate Composite Materials)</u>

Those consist of layers (plies) of various materials

<u>Lamina (ply): -</u> A composite made by single layer of material. Usually a flat arrangement of unidirectional fibers or woven fibers in matrix.

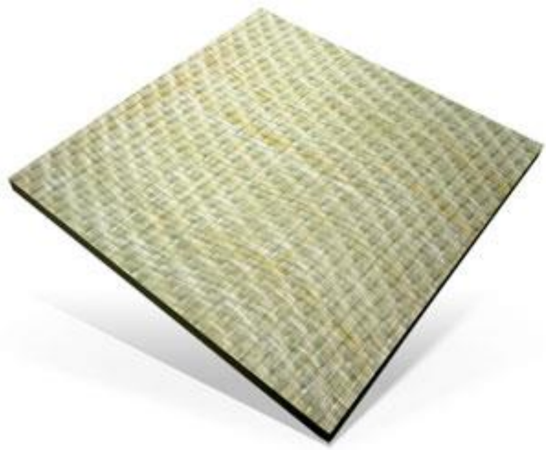


Figure 1 woven

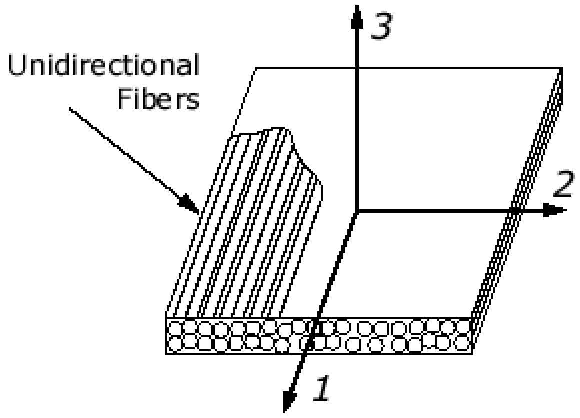


Figure 2 unidirectional

<u>Laminate</u>: - A material made by bonding together a series of lamiae. Plywood is an example where thin sheets of wood are bonded together to give a stronger laminated structure.

<u>2.1 - Application</u>:-

(i) Bimetal e.g. simple thermostat (temperature indicator)

For single homogenous strip, if T ↑ ⟹ extension only

- for bimetals if T↑ ⟹ coupling behavior bending and extension .

By measuring the curvature or deflection the strip can turn on or turn off a furnace or air conditioner.

<u>Bimetallic</u>: - A laminate composite material produced by joining two strips of metal with different thermal expansion coefficient.

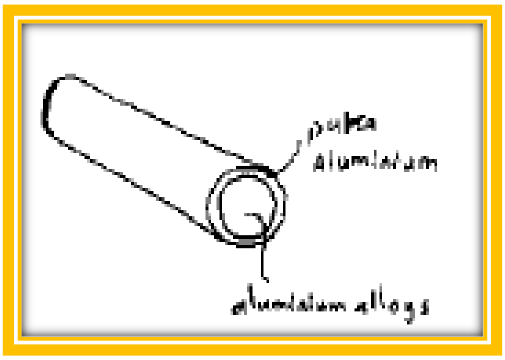
(ii) Clad Metals

Clad materials are:-

Metal- metal composites clad materials.

Provide combination of good corrosion

resistance with high strength.

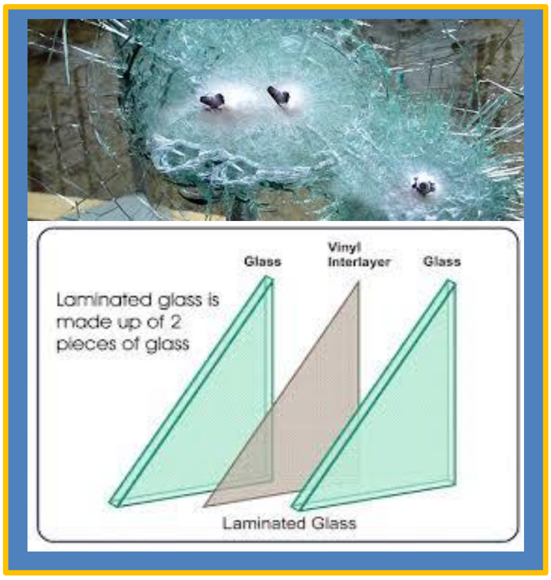


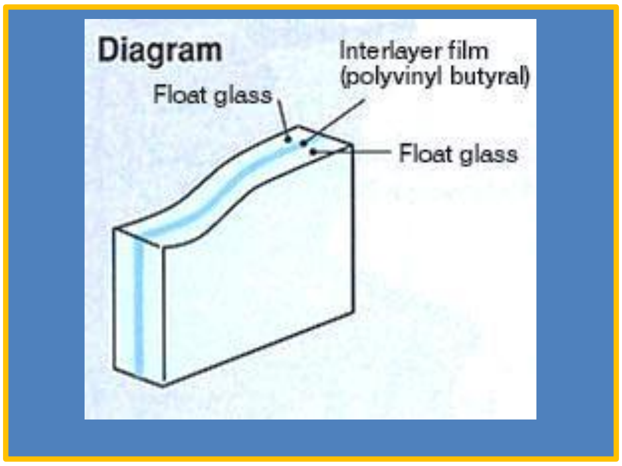
Aluminum alloy are not very corrosive resistance, where as pure aluminum is so, a cladding of pure aluminum over a high strength aluminum alloys is a composite material with better properties.

The thickness of the pure aluminum layer is about (1% to 15%) of the total thickness, is used in a storage tank, aircraft construction and heat exchangers.

(iii) Laminated Glass e.g. " safely glass "

Due to plastic the deforms occurs to high strain without fracture.





(iv) Laminated fibrous composite

These involve fibrous composites and lamination techniques.

They are commonly termed laminated fiber reinforced composites. e.g;

- Fiber glass boat hulls.
- Aircraft wing panels and body section…….. etc.

2.2 - Rule of mixtures:- some properties of the laminar composite materials in the longitudinal direction are estimated from the rule of mixtures . The density, electrical and thermal conductivity, and modulus of elasticity **parallel to the laminae**.

Density =$\boldsymbol{\rho}_{c,II} = \sum \boldsymbol{\rho}_{\boldsymbol{i}} * \boldsymbol{V}_{\boldsymbol{i}}$

Electrical conductivity = $\boldsymbol{\sigma}_{c,II} = \sum \boldsymbol{\sigma}_{\boldsymbol{i}} * \boldsymbol{V}_{\boldsymbol{i}}$

Thermal conductivity = $\boldsymbol{K}_{C,II} = \sum \boldsymbol{K}_{\boldsymbol{i}} * \boldsymbol{V}_{\boldsymbol{i}}$

Modulus of elasticity = $\boldsymbol{E}_{C,II} = \sum \boldsymbol{E}_{\boldsymbol{i}} * \boldsymbol{V}_{\boldsymbol{i}}$

While the **properties perpendicular to the laminae**

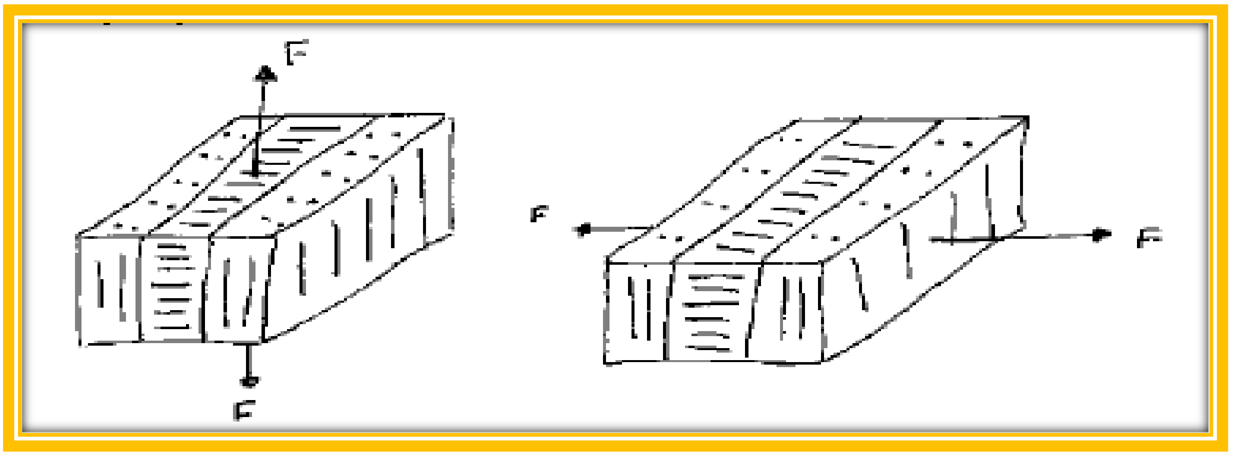
Electrical conductivity = $\frac{\mathbf{1}}{\boldsymbol{\sigma}_{\boldsymbol{c},\perp}} = \sum \frac{\boldsymbol{V}_{\boldsymbol{i}}}{\boldsymbol{\sigma}_{\boldsymbol{i}}}$

Thermal conductivity = $\frac{\mathbf{1}}{\boldsymbol{K}_{\boldsymbol{c},\perp}} = \sum \frac{\boldsymbol{V}_{\boldsymbol{i}}}{\boldsymbol{K}_{\boldsymbol{i}}}$

Modulus of elasticity= $\frac{\mathbf{1}}{\boldsymbol{E}_{\boldsymbol{c},\perp}} = \sum \frac{\boldsymbol{V}_{\boldsymbol{i}}}{\boldsymbol{E}_{\boldsymbol{i}}}$

Example:

A sheet of plywood consists of three equally thickness sheets, the left and right sheets having their fiber in the same direction and the middle sheet with its fiber at right angle. The wood has a tensile modulus for forces in the direction parallel to the fiber of (10 GPa), and in the transverse direction (0.4 GPa). Determine the tensile modulus of the laminate when loaded in a direction parallel to the fiber direction of the outer sheet and the tensile modulus of the laminate when loaded in a directional perpendicular to the fiber direction of the outer sheet.



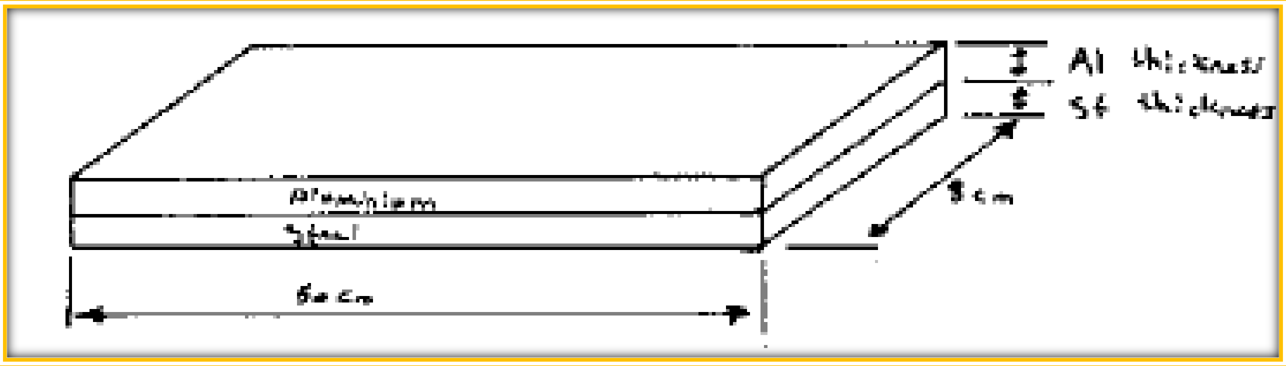
Solution:-

$$E_{c,II} = E_1 * V_1 + E_2 * V_2 + E_3 * V_3$$

$$= 10*\frac{1}{3} + 0.4 * \frac{1}{3} + 10 * \frac{1}{3} = 6.8\ GPa$$

$$\frac{1}{E_{C,\perp}} = \frac{V_1}{E_1} + \frac{V_2}{E_2} + \frac{V_3}{E_3} = \frac{\frac{1}{3}}{10} + \frac{\frac{1}{3}}{0.4} + \frac{\frac{1}{3}}{10} = 1.11\ \ GPa$$

Example: for the following composite beam (multi-layer beam)



Determine the modulus of elasticity of the composite beam in both directions at the following conditions

AL thickness (cm)	St thickness (cm)
1	1
0.5	1.5
1.5	0.5

Note :-

E_{st} = 207 GPa

E_{al}= 68.6 GPa

And compare the results of the composite beam.

Solution:-

at V_{Al} = 0.5 and V_{St}= 0.5

$$E_{c,II} = E_{Al} * V_{Al} + E_{St} * V_{St} = 68.6 * 0.5 + 207 * 0.5 = 137.8 \text{ GPa}$$

$$E_{C\perp} = \frac{68.6 * 207}{207 * 0.5 + 68.8 * 0.5} = 103.1\ GPa$$

at V_{Al} = 0.25 and V_{St}= 0.75

$$E_{cII} = E_{Al} * V_{Al} + E_{St} * V_{St} = 68.6 * 0.25 + 207 * 0.75 = 172.24$$
GPa

$$E_{C\perp} = \frac{E_{St} * E_{Al}}{E_{Al} * V_{Al} + E_{St} * V_{St}} = \frac{207 * 68.6}{207 * 0.25 + 68.8 * 0.75}$$

$$= 137.57\ GPa$$

at V_{Al} = 0.75 and V_{St}= 0.25

$$E_{cII} = E_{Al} * V_{Al} + E_{St} * V_{St} = 68.6 * 0.75 + 207 * 0.25 = 103.2 \text{ GPa}$$

$$E_{C\perp} = \frac{E_{St} * E_{Al}}{E_{St} * V_{Al} + E_{Al} * V_{St}} = \frac{207 * 68.6}{207 * 0.75 + 68.8 * 0.25}$$

$$= 82.37\ GPa$$

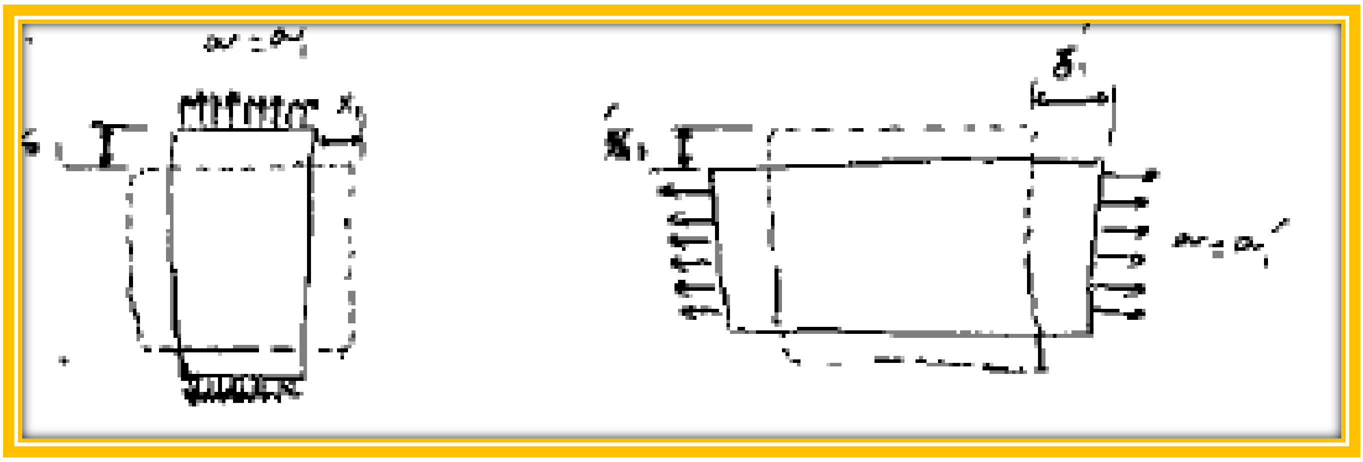
Steel thickness (cm)	Aluminum thickness (cm)	$E_{c,II}$ (GPa)	$E_{C,\perp}$ (GPa)
2	0	207	207
1.5	0.5	172.24	137.57
1	1	137.8	103.1
0.5	1.5	103.2	82.37
0	2	68.6	68.6

2.3 – Homogeneous body and Isotropic

Composite materials are different form engineering material most common engineering materials are homogeneous and isotropic.

A Homogeneous body:- material properties (stiffness, strength ,…….etc.) remain constant form point to point in a direction in the body, i.e.(The properties are not a function at a point in the body).

An Isotropic Material :- has material properties that are the same in every direction at a point in the body .



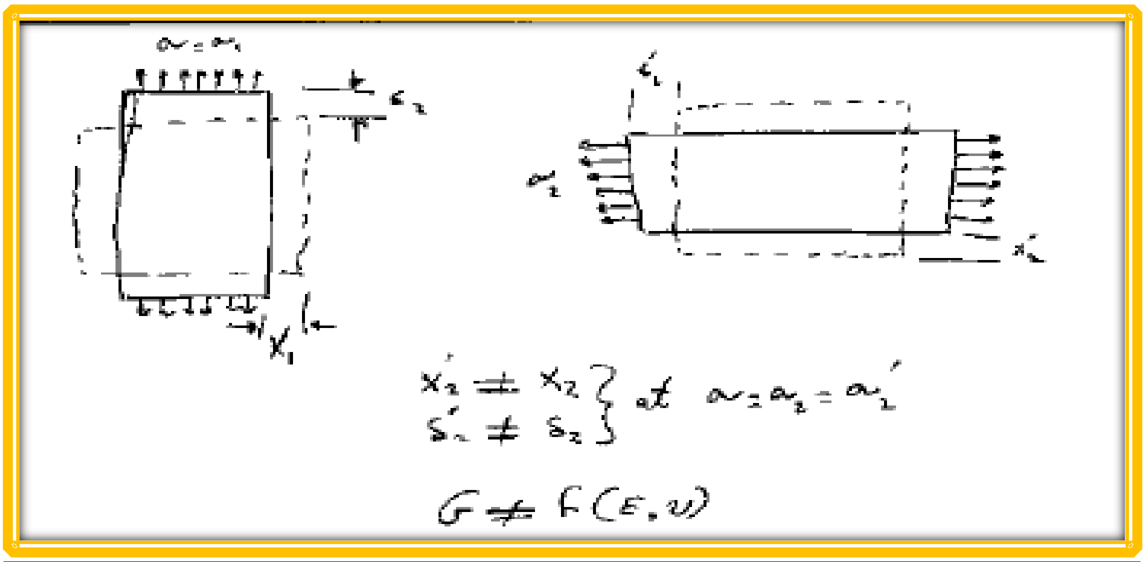
$$\left.\begin{aligned} x_1' &= x_1 \\ \delta_1' &= \delta_1 \end{aligned}\right\} \quad \text{at } \sigma = \sigma_1 = \sigma_1' \qquad G = f(E,\nu) = \frac{E}{2(1+\nu)}$$

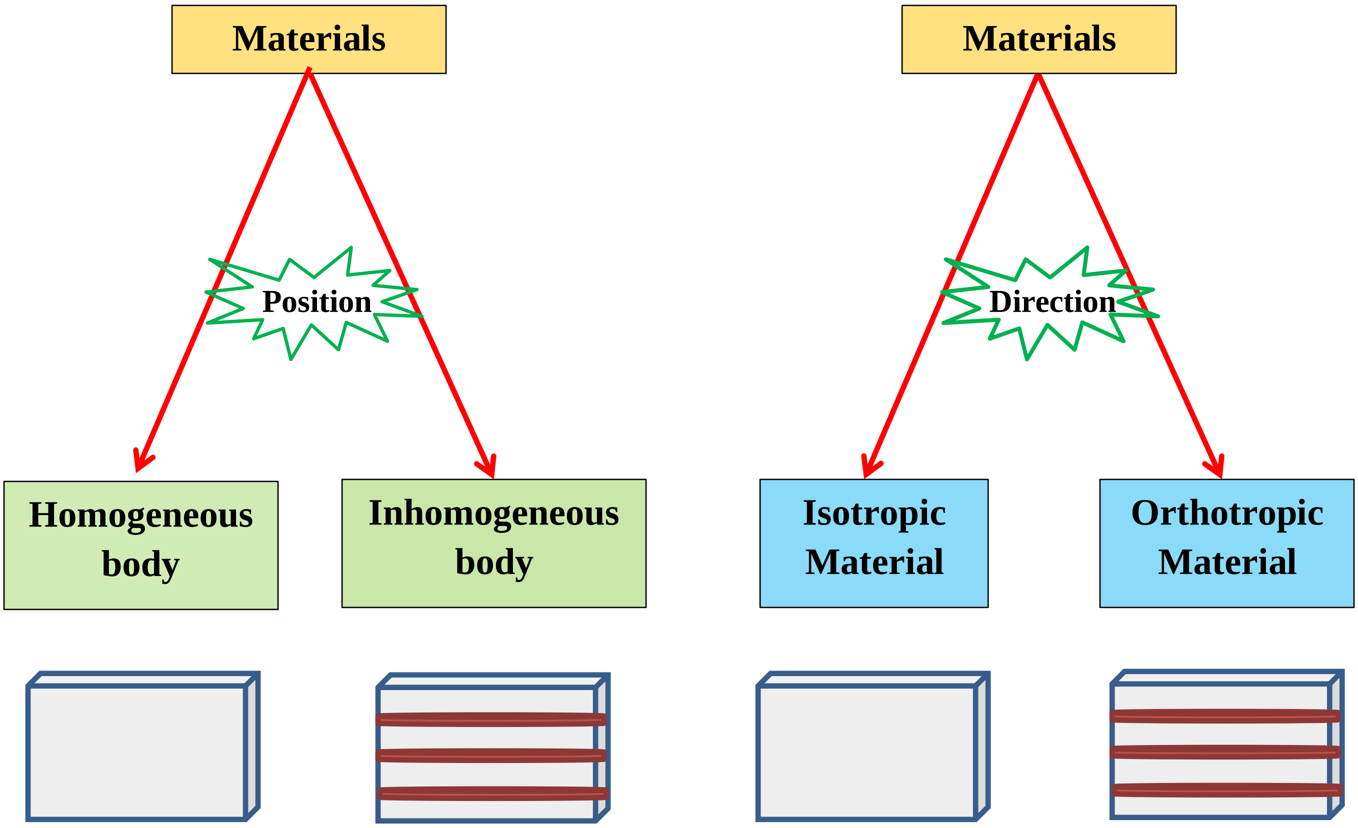
While composite materials are inhomogeneous (heterogeneous) and non-isotropic (orthotropic and anisotropic).

2.4 – Inhomogeneous body and Orthotropic

An Inhomogeneous body: - has non uniform properties over the body. i.e. the properties are a function of position in the body.

An Orthotropic Material: - has material properties that are different in three mutually perpendicular directions at a point of the body.




Materials
Position
Homogeneous body
Inhomogeneous body
Materials
Direction
Isotropic Material
Orthotropic Material

Tutorial 1 (مناقشة 1)

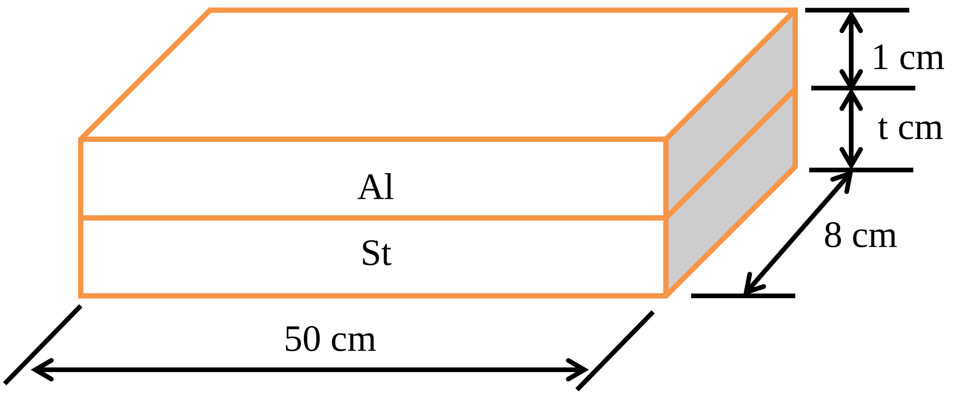
Q1) For the following multi-layer beam determine the thickness (t) of the second layer.

Note:

E_{st}= 207 GPa.

E_{Al}= 69 GPa.

$E_{c,II}$= 138 GPa.



Solution:

$$E_{c,II} = E_{Al} * V_{Al} + E_{st} * V_{st}$$

$$138 = 69 * V_{Al} + 207 * V_{st}$$

$$V_{Al} = \frac{1 * 8 * 50}{(1+t) * 8 * 50} = \frac{1}{1+t}$$

$$V_{st} = \frac{t * 8 * 50}{(1+t) * 8 * 50} = \frac{t}{1+t}$$

$$138 = 69 * \frac{1}{(1+t)} + 207 * \frac{t}{(1+t)}$$

$$138 * (1+t) = 69 + 207 * t$$

$$138 + 138t = 69 + 207 * t$$

$$207 * t - 138 * t = 138 - 69$$

$$\therefore t = \frac{69}{69} = 1\ cm$$

Generally laminate layers are bonded by the same matrix used in the plies

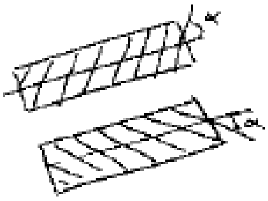
symmetric laminates

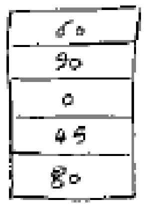
antisymmetric laminates

nonsymmetrical

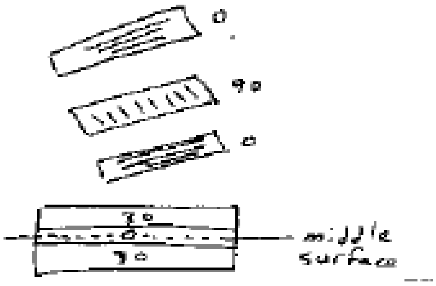
laminates

with isotropic layer





with orthotropic layer



Also the properties of a unidirectional lamina are found by using the following equations:-

$$E_1 = E_f * V_f + E_m * V_m$$

$$E_2 = \frac{E_f * E_m}{E_f * V_m + E_m * V_f}$$

$$G_{12} = \frac{G_f * G_m}{G_f * V_m + G_m * V_f} \qquad G_f = \frac{E_f}{2(1+\nu_f)}$$

$$G_m = \frac{E_m}{2(1+\nu_m)}$$

$$\nu_{12} = \nu_f * V_f + \nu_m * V_m$$

$$\frac{E_1}{\nu_{12}} = \frac{E_2}{\nu_{21}}$$ (Law of Maxwell's theorem)

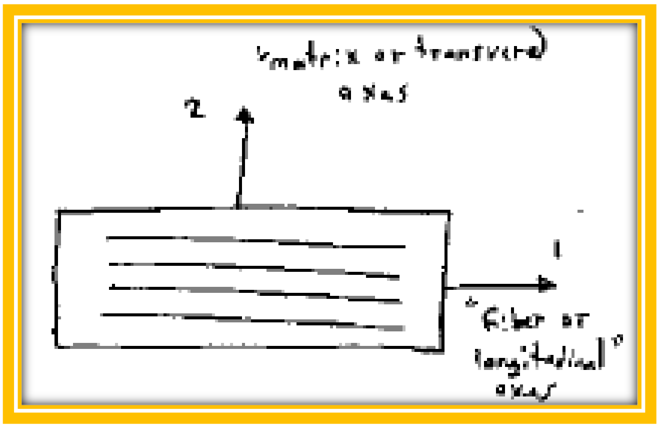
2.5 Axes of structural composite:-

Materials axes
Principal axes
Symmetry axes

A set of mutually perpendicular directions parallel and perpendicular to the fiber direction

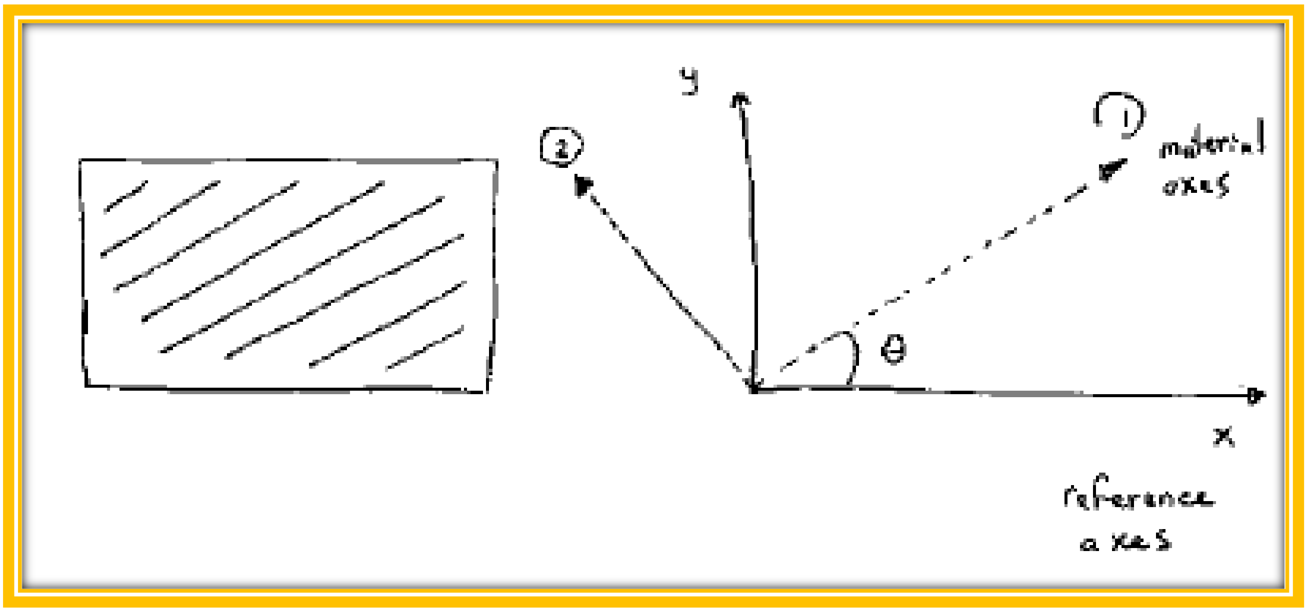
$$E_x = E_1 = E_m * V_m + E_f * V_f$$

$$\frac{1}{E_y} = \frac{1}{E_2} = \frac{V_m}{E_m} + \frac{V_f}{E_f}$$



Reference axes
Structure axes
Loading axes

A set of mutually perpendicular directions parallel and perpendicular to the reference direction generally coincident with the external loading system axes



$$E_x = f(E_1\ E_2\ G_{12}\ \nu_{12}\ \theta)$$

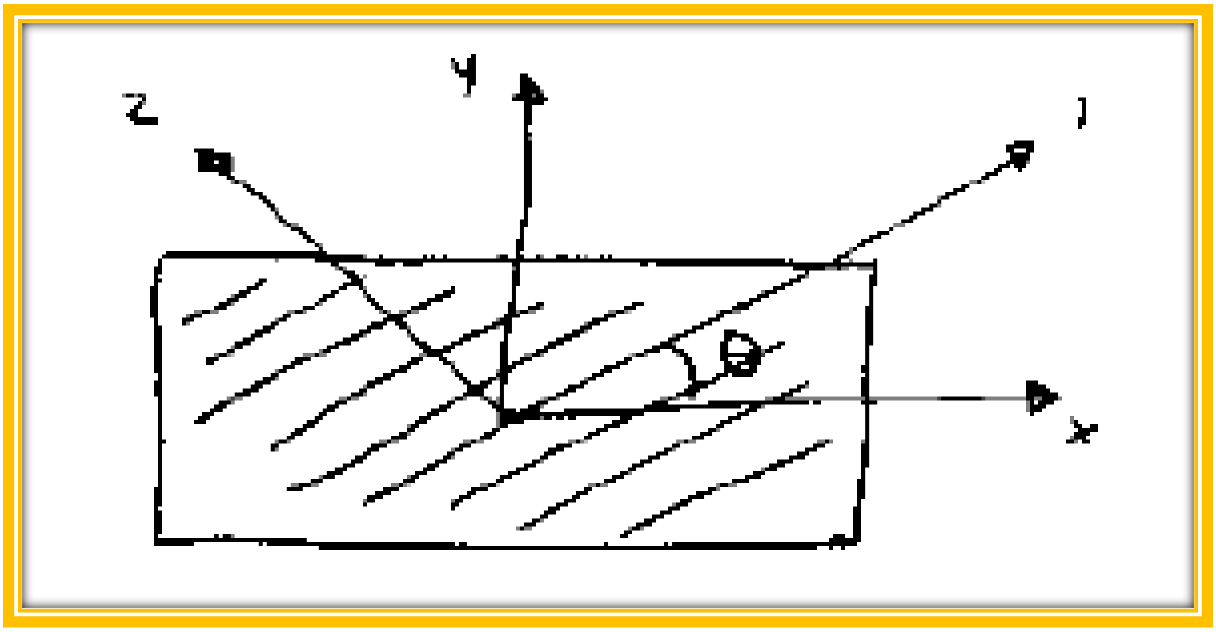
$$E_y = f(E_1\ E_2\ G_{12}\ \nu_{12}\ \theta)$$

3 - Transformation of Engineering Elastic Constants:

From the definition of an orthotropic material, the composite ply will have different properties in different directions at a point.

Now need to know how (E_1 E_2 G_{12} ν_{12} θ) change with the axes directions.

The elastic moduli for an angle lamina are given as:



$$\frac{1}{E_x} = \frac{\cos^4\theta}{E_1} + \frac{\sin^4\theta}{E_2} + \left(\frac{1}{G_{12}} - \frac{2\nu_{12}}{E_1}\right) * \sin^2\theta * \cos^2\theta$$

And

$$\nu_{xy} = E_x[\frac{\nu_{12}}{E_1}(\sin^4\theta + \cos^4\theta) - \left(\frac{1}{E_1} + \frac{1}{E_2} - \frac{1}{G_{12}}\right)\sin^2\theta * \cos^2\theta]$$

$$\frac{1}{E_y} = \frac{\sin^4\theta}{E_1} + \frac{\cos^4\theta}{E_2} + \left(\frac{1}{G_{12}} - \frac{2\nu_{12}}{E_1}\right) * \sin^2\theta * \cos^2\theta$$

$$\nu_{yx} = E_y[\frac{\nu_{12}}{E_1}(\sin^4\theta + \cos^4\theta) - \left(\frac{1}{E_1} + \frac{1}{E_2} - \frac{1}{G_{12}}\right)\sin^2\theta * \cos^2\theta]$$

$$\frac{1}{G_{xy}} = \left(\frac{4}{E_1} + \frac{4}{E_2} + \frac{8\nu_{12}}{E_1}\right) * \cos^2\theta * sin^2\theta + \frac{(cos^2\theta - sin^2\theta)^2}{G_{12}}$$

Example :-

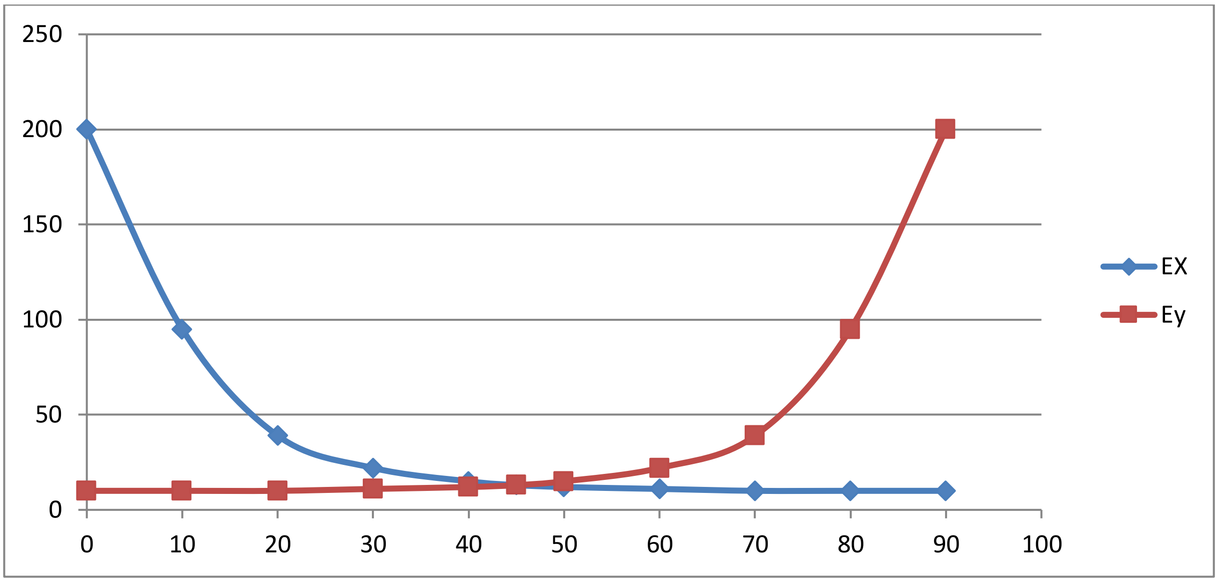
for the data given

E_1=200MPa E_2=10MPa G_{12}=5MPa ν_{12}=0.3

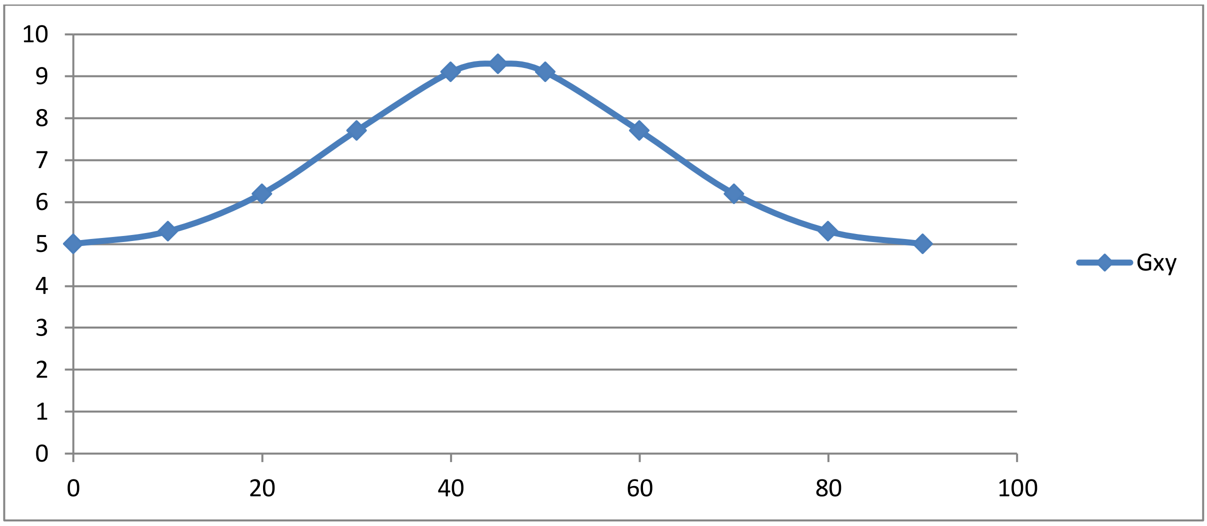
Calculate and draw the variation of equivalent elastic properties with the angle of fiber i.e. $(E_x, E_y, G_{xy}, \nu_{xy}, \nu_{yx})$

Solution:

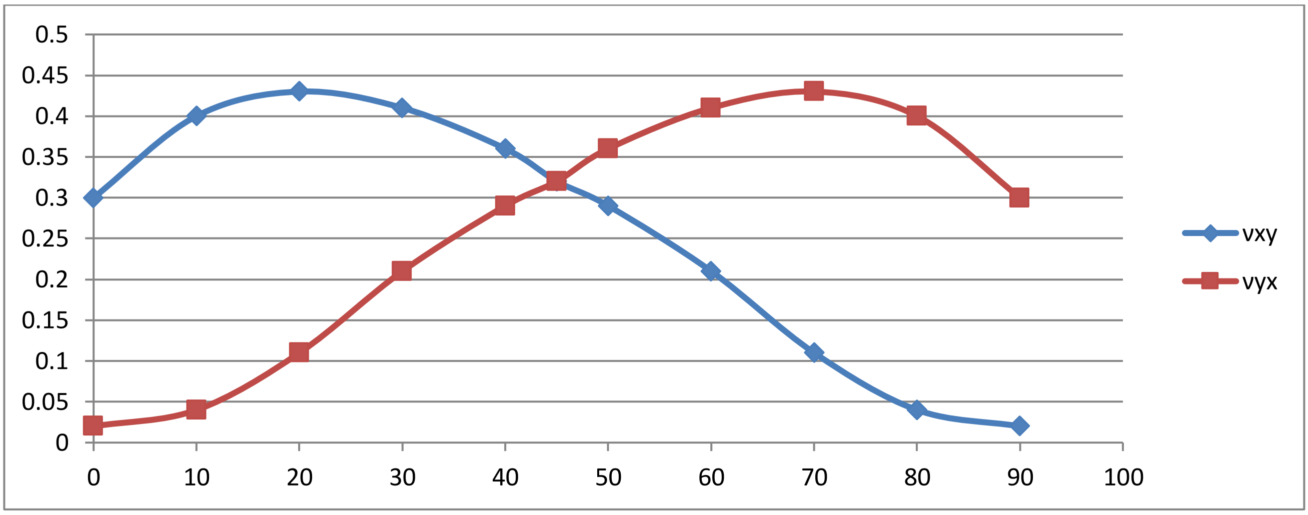
θ^0	E_x	E_y	G_{xy}	ν_{xy}	ν_{yx}
0	200	10	5	0.3	0.02
10	95	10	5.3	0.4	0.04
20	39	10	6.2	0.43	0.11
30	22	11	7.7	0.41	0.21
40	15	12	9.1	0.36	0.29
45	13	13	9.3	0.32	0.32
50	12	15	9.1	0.29	0.36
60	11	22	7.7	0.21	0.41
70	10	39	6.2	0.11	0.43
80	10	95	5.3	0.04	0.4
90	10	200	5	0.02	0.3



250
200
150
100
50
0
0
10
20
30
40
50
60
70
80
90
100
EX
Ey



10
9
8
7
6
5
4
3
2
1
0
0
20
40
60
80
100
Gxy



0.5
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
0
10
20
30
40
50
60
70
80
90
100
vxy
vyx

The general observation:-

1- When θ = 0^0, i.e. specially orthotropic ply

$E_x = E_1$ $\qquad$ $E_y = E_2$

$\nu_{xy} = \nu_{12}$ $\qquad$ $\nu_{yx} = \nu_{21}$ $\qquad$ $G_{xy} = G_{12}$

2- When θ = 90^0, i.e. specially orthotropic ply

$E_x = E_2$ $\qquad$ $E_y = E_1$

$\nu_{xy} = \nu_{21}$ $\qquad$ $\nu_{yx} = \nu_{12}$

3- When θ = 45^0

G_{xy} is largest

$E_x = E_y$

$\nu_{xy} = \nu_{yx}$

Example: **for the following data:**

E_1= 100GPa $\qquad$ E_2=10GPa $\qquad$ G_{12}= 5GPa

$\nu_{12} = 0.29$ $\quad$ *and* $\quad$ θ = 45^0

Calculate the equivalent elastic constants $(\boldsymbol{E_x}\ \boldsymbol{\nu_{xy}}\ \boldsymbol{E_y}\ \boldsymbol{\nu_{yx}}\ \boldsymbol{G_{xy}})$.