Thermal conductivity

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Heat Capacity

The ability of a material to absorb heat

- Quantitatively: The energy required to produce a unit rise in temperature for one mole of a material.

$$C = \frac{dQ}{dT}$$

heat capacity (J/mol-K) → C

energy input (J/mol) → dQ

temperature change (K) → dT

- Two ways to measure heat capacity:

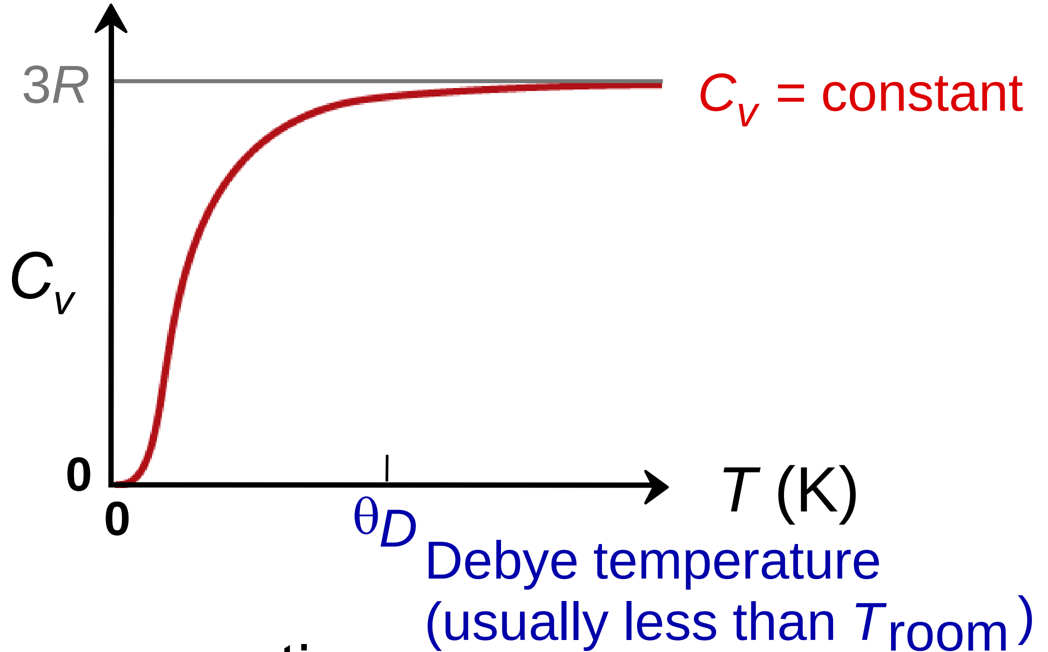
  C_p : Heat capacity at constant pressure.
  C_v : Heat capacity at constant volume.

  C_p usually > C_v

Dependence of Heat Capacity on Temperature

- Heat capacity...
  -- increases with temperature
  -- for solids it reaches a limiting value of 3*R*

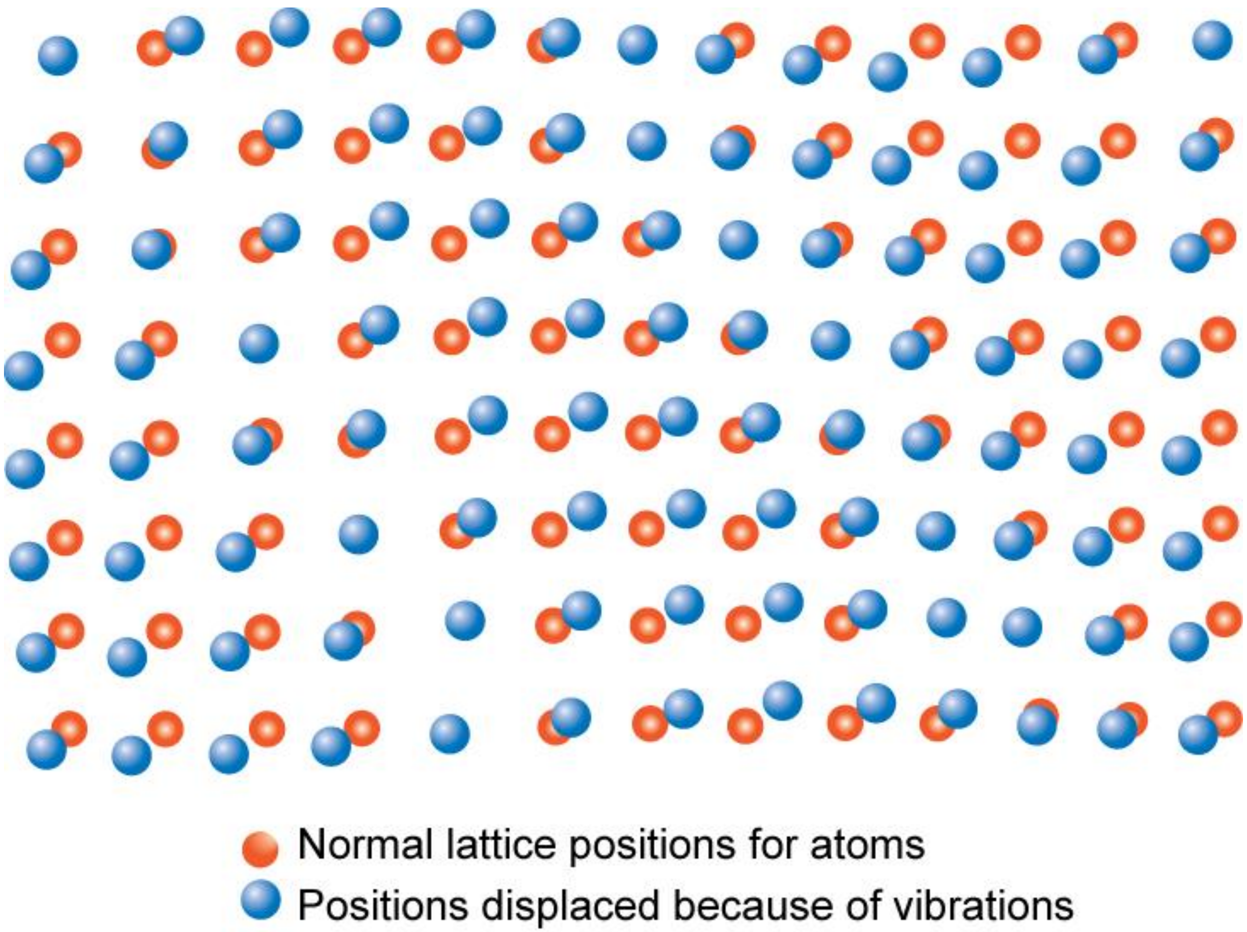
R = gas constant
= 8.31 J/mol-K



- From atomic perspective:
  -- Energy is stored as atomic vibrations.
  -- As temperature increases, the average energy of atomic vibrations increases.

Atomic Vibrations

Atomic vibrations are in the form of lattice waves or phonons



Specific Heat: Comparison

increasing c_p

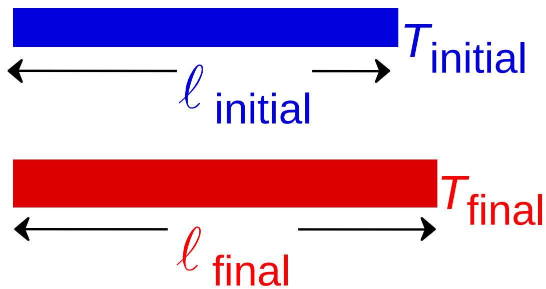
Material	c_p (J/kg-K) at room T
• <u>Polymers</u>	
Polypropylene	1925
Polyethylene	1850
Polystyrene	1170
Teflon	1050
• <u>Ceramics</u>	
Magnesia (MgO)	940
Alumina (Al_2O_3)	775
Glass	840
• <u>Metals</u>	
Aluminum	900
Steel	486
Tungsten	138
Gold	128

c_p (specific heat): (J/kg-K)

C_p (heat capacity): (J/mol-K)

Thermal Expansion

Materials change size when temperature is changed

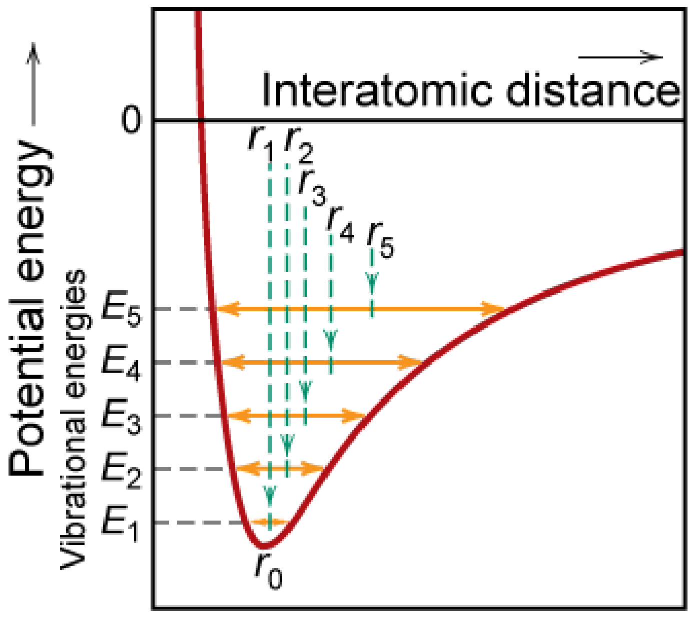


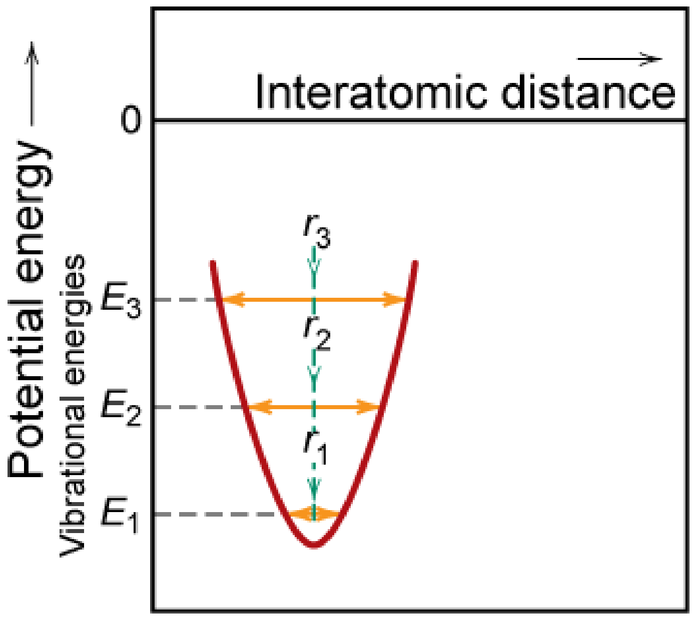
$$T_{\text{final}} > T_{\text{initial}}$$

$$\frac{\ell_{\text{final}} - \ell_{\text{initial}}}{\ell_{\text{initial}}} = \alpha_{\ell}\left(T_{\text{final}} - T_{\text{initial}}\right)$$

linear coefficient of thermal expansion (1/K or 1/°C)

Atomic Perspective: Thermal Expansion





Asymmetric curve:
-- increase temperature,
-- increase in interatomic departure
-- thermal expansion

Symmetric curve:
-- increase temperature,
-- no increase in interatomic departure
-- no thermal expansion

Coefficient of Thermal Expansion: Comparison

increasing α_ℓ

Material	α_ℓ (10^{-6}/°C) at room *T*
• Polymers	
Polypropylene	145-180
Polyethylene	106-198
Polystyrene	90-150
Teflon	126-216
• Metals	
Aluminum	23.6
Steel	12
Tungsten	4.5
Gold	14.2
• Ceramics	
Magnesia (MgO)	13.5
Alumina (Al_2O_3)	7.6
Soda-lime glass	9
Silica (cryst. SiO_2)	0.4

Polymers have larger α_ℓ values because of weak secondary bonds

Thermal Conductivity

The ability of a material to transport heat.

Fourier's Law

$$q = -k\frac{dT}{dx}$$

heat flux (J/m^2-s) → q

temperature gradient → $\frac{dT}{dx}$

thermal conductivity (J/m-K-s) → k

$T_2 > T_1$

- Atomic perspective: Atomic vibrations and free electrons in hotter regions transport energy to cooler regions.

Thermal Conductivity: Comparison

increasing k

Material	k (W/m-K)	Energy Transfer Mechanism
• Metals		
Aluminum	247	atomic vibrations and motion of free electrons
Steel	52	
Tungsten	178	
Gold	315	
• Ceramics		
Magnesia (MgO)	38	atomic vibrations
Alumina (Al_2O_3)	39	
Soda-lime glass	1.7	
Silica (cryst. SiO_2)	1.4	
• Polymers		
Polypropylene	0.12	vibration/rotation of chain molecules
Polyethylene	0.46-0.50	
Polystyrene	0.13	
Teflon	0.25	

Thermal Stresses

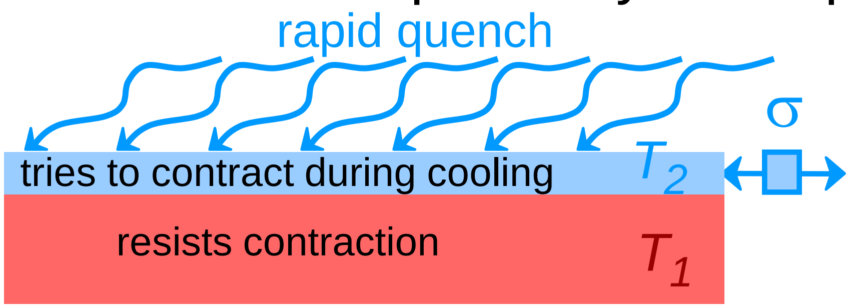
- Occur due to:
  - -- restrained thermal expansion/contraction
  - -- temperature gradients that lead to differential dimensional changes

Thermal stress $= \sigma$

$$= E\alpha_{\ell}(T_0 - T_f) = E\alpha_{\ell}\Delta T$$

Thermal Shock Resistance

- Occurs due to: nonuniform heating/cooling
- Ex: Assume top thin layer is rapidly cooled from T_1 to T_2



Tension develops at surface

$$\sigma = -E\alpha_\ell (T_1 - T_2)$$

Temperature difference that can be produced by cooling:

$$(T_1 - T_2) = \frac{\text{quench rate}}{k}$$

Critical temperature difference for fracture (set $\sigma = \sigma_f$)

$$(T_1 - T_2)_{\text{fracture}} = \frac{\sigma_f}{E\alpha_\ell}$$

set equal

- $(\text{quench rate})_{\text{for fracture}} = \text{Thermal Shock Resistance } (TSR) \propto \dfrac{\sigma_f k}{E\alpha_\ell}$
- Large TSR when $\dfrac{\sigma_f k}{E\alpha_\ell}$ is large

Summary

The thermal properties of materials include:

- **Heat capacity:**
  -- energy required to increase a mole of material by a unit *T*
  -- energy is stored as atomic vibrations
- **Coefficient of thermal expansion:**
  -- the size of a material changes with a change in temperature
  -- polymers have the largest values
- **Thermal conductivity:**
  -- the ability of a material to transport heat
  -- metals have the largest values
- **Thermal shock resistance:**
  -- the ability of a material to be rapidly cooled and not fracture
  -- is proportional to $\frac{\sigma_f k}{E\alpha_\ell}$

Example

Ex: A copper wire 15 m long is cooled from 40 to -9°C. How much change in length will it experience?

- Answer: For Cu $\alpha_\ell = 16.5 \text{ x } 10^{-6} \ (^\circ\text{C})^{-1}$

rearranging Equation 17.3b

$$\Delta\ell = \alpha_\ell \ell_0 \Delta T = [16.5 \text{ x } 10^{-6} (1/{}^\circ\text{C})](15 \text{ m})[40^\circ\text{C} - (-9^\circ\text{C})]$$

$$\Delta\ell = 0.012 \text{ m} = 12 \text{ mm}$$

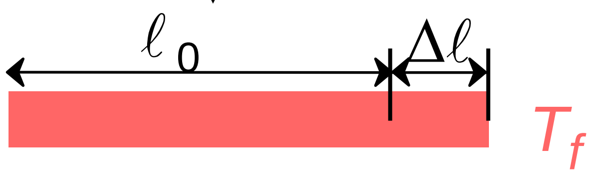
Example2

-- A brass rod is stress-free at room temperature (20°C).
-- It is heated up, but prevented from lengthening.
-- At what temperature does the stress reach -172 MPa?

Solution:

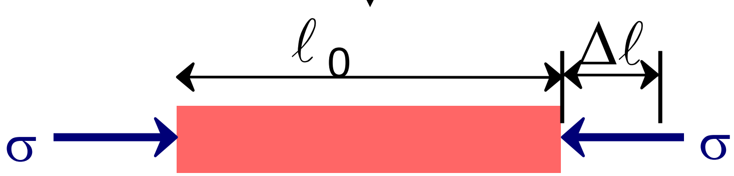
Original conditions

Step 1: Assume unconstrained thermal expansion



$$\frac{\Delta\ell}{\ell_{room}} = \varepsilon_{thermal} = \alpha_\ell (T_f - T_0)$$

Step 2: Compress specimen back to original length



$$\varepsilon_{compress} = \frac{-\Delta\ell}{\ell_{room}} = -\varepsilon_{thermal}$$