

# Phase Diagrams

- When we combine two elements...  
    what equilibrium state do we get?
- In particular, if we specify...
  - a composition (e.g., wt% Cu - wt% Ni), and
  - a temperature ( $T$ )then...
  - How many phases do we get?
  - What is the composition of each phase?
  - How much of each phase do we get?

# Phase Equilibria

Simple solution system (e.g., Ni-Cu solution)

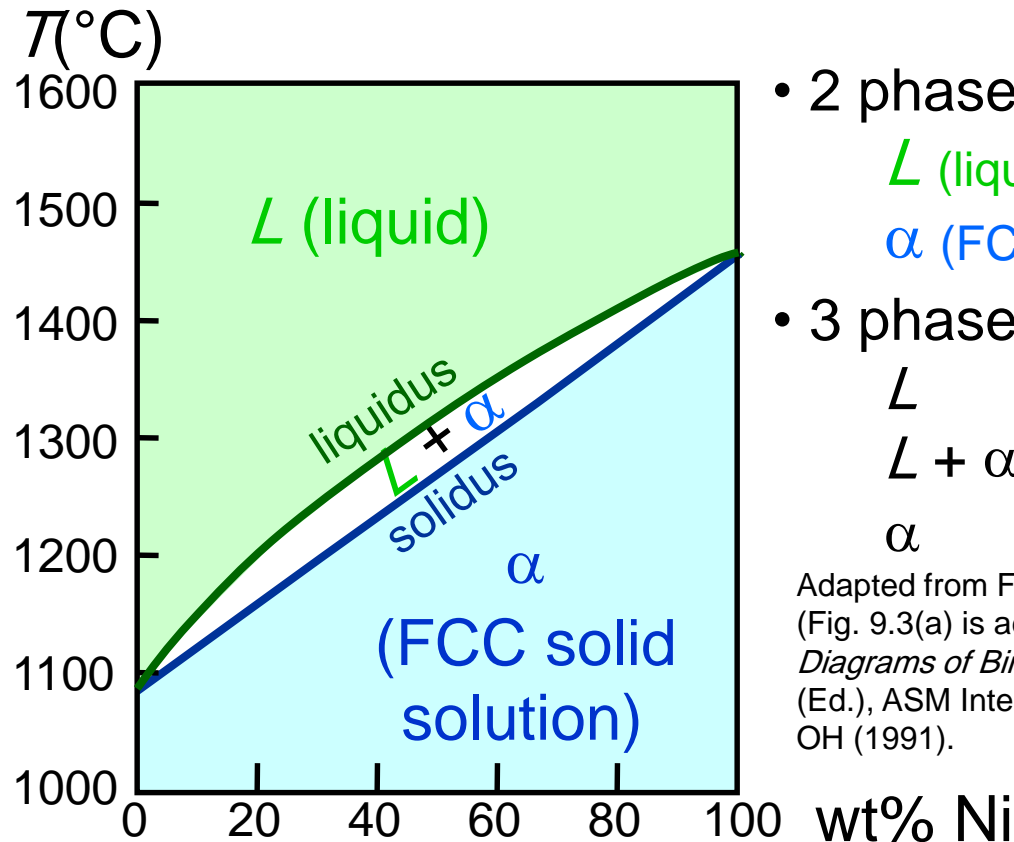
	Crystal Structure	electroneg	$r$ (nm)
Ni	FCC	1.9	0.1246
Cu	FCC	1.8	0.1278

- Both have the same crystal structure (FCC) and have similar electronegativities and atomic radii ([W. Hume – Rothery rules](#)) suggesting high mutual solubility.
- Ni and Cu are totally miscible in all proportions.

# Phase Diagrams

- Indicate phases as function of  $T$ ,  $C_o$ , and  $P$ .
- For this course:
  - binary systems: just 2 components.
  - independent variables:  $T$  and  $C_o$  ( $P = 1$  atm is almost always used).

- Phase Diagram for Cu-Ni system



- 2 phases:
  - $L$  (liquid)
  - $\alpha$  (FCC solid solution)
- 3 phase fields:
  - $L$
  - $L + \alpha$
  - $\alpha$

Adapted from Fig. 9.3(a), *Callister 7e*.  
(Fig. 9.3(a) is adapted from *Phase Diagrams of Binary Nickel Alloys*, P. Nash (Ed.), ASM International, Materials Park, OH (1991).

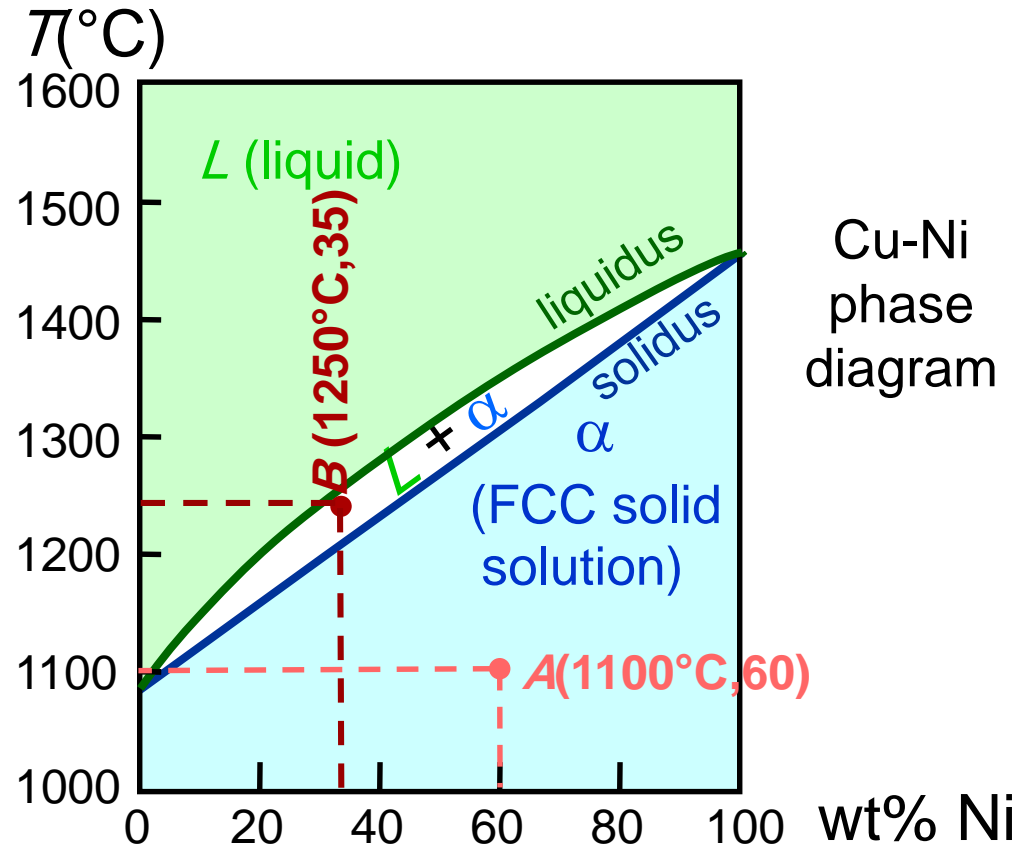
# Phase Diagrams: # and types of phases

- Rule 1: If we know  $T$  and  $C_0$ , then we know:  
--the # and types of phases present.

- Examples:

$A(1100^\circ\text{C}, 60)$ :  
1 phase:  $\alpha$

$B(1250^\circ\text{C}, 35)$ :  
2 phases:  $L + \alpha$



Adapted from Fig. 9.3(a), *Callister 7e*.  
(Fig. 9.3(a) is adapted from *Phase Diagrams of Binary Nickel Alloys*, P. Nash (Ed.), ASM International, Materials Park, OH, 1991).

# Phase Diagrams: composition of phases

- Rule 2: If we know  $T$  and  $C_0$ , then we know:  
--the composition of each phase.

- Examples:

$$C_0 = 35 \text{ wt\% Ni}$$

At  $T_A = 1320^\circ\text{C}$ :

Only Liquid ( $L$ )

$$C_L = C_0 (= 35 \text{ wt\% Ni})$$

At  $T_D = 1190^\circ\text{C}$ :

Only Solid ( $\alpha$ )

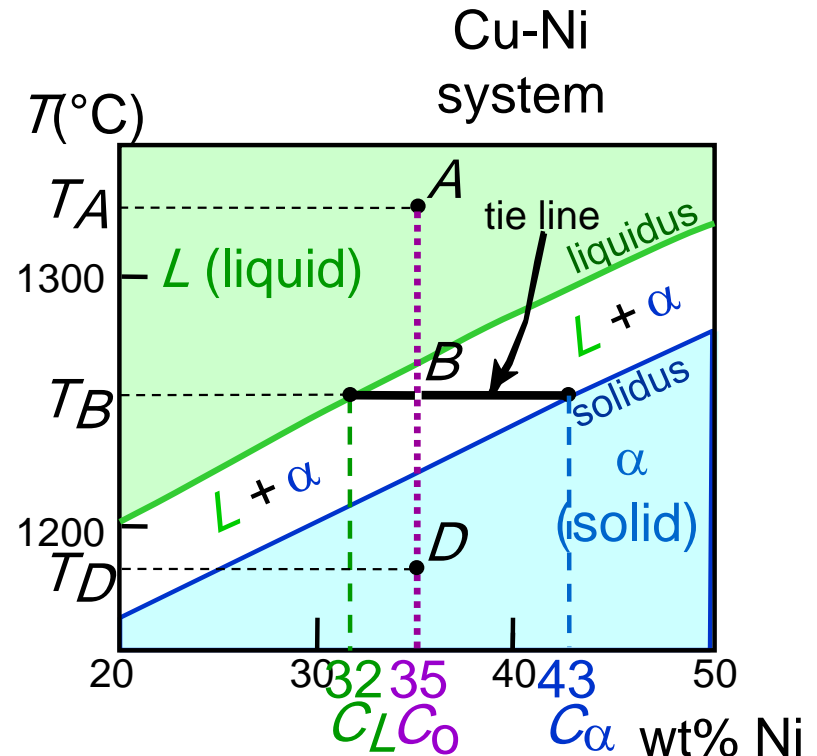
$$C_\alpha = C_0 (= 35 \text{ wt\% Ni})$$

At  $T_B = 1250^\circ\text{C}$ :

Both  $\alpha$  and  $L$

$$C_L = C_{\text{liquidus}} (= 32 \text{ wt\% Ni here})$$

$$C_\alpha = C_{\text{solidus}} (= 43 \text{ wt\% Ni here})$$



Adapted from Fig. 9.3(b), *Callister 7e*.  
(Fig. 9.3(b) is adapted from *Phase Diagrams of Binary Nickel Alloys*, P. Nash (Ed.), ASM International, Materials Park, OH, 1991.)

# Phase Diagrams: weight fractions of phases

- Rule 3: If we know  $T$  and  $C_o$ , then we know:
  - the amount of each phase (given in wt%).

- Examples:

$$C_o = 35 \text{ wt\% Ni}$$

At  $T_A$ : Only Liquid (L)

$$W_L = 100 \text{ wt\%}, W_\alpha = 0$$

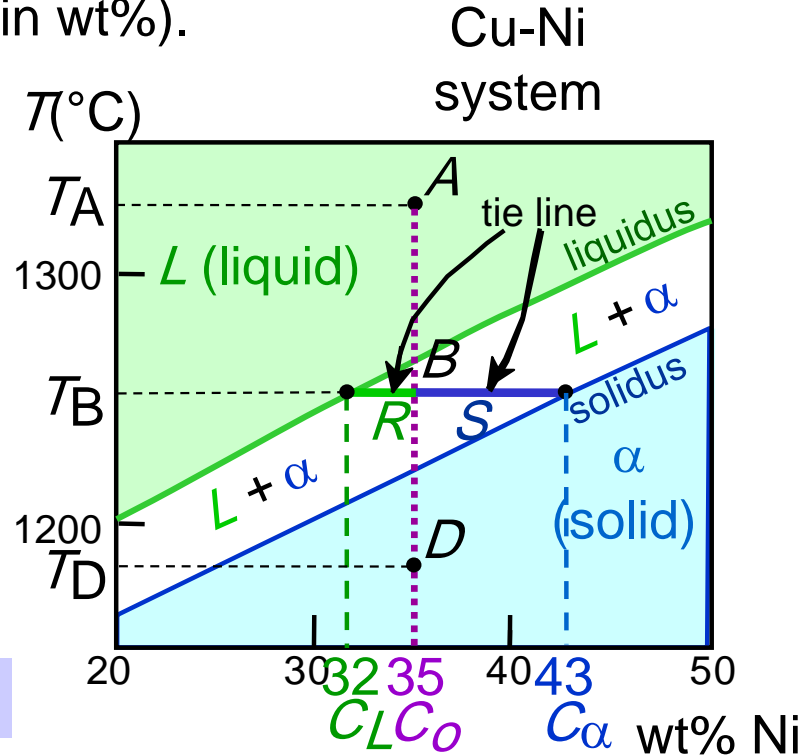
At  $T_D$ : Only Solid ( $\alpha$ )

$$W_L = 0, W_\alpha = 100 \text{ wt\%}$$

At  $T_B$ : Both  $\alpha$  and L

$$W_L = \frac{S}{R+S} = \frac{43 - 35}{43 - 32} = 73 \text{ wt\%}$$

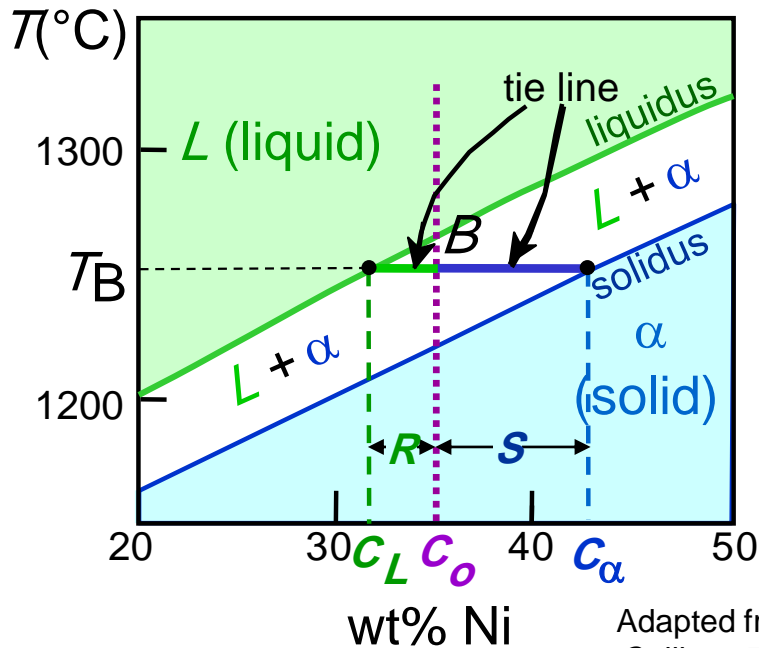
$$W_\alpha = \frac{R}{R+S} = 27 \text{ wt\%}$$



Adapted from Fig. 9.3(b), *Callister 7e*.  
(Fig. 9.3(b) is adapted from *Phase Diagrams of Binary Nickel Alloys*, P. Nash (Ed.), ASM International, Materials Park, OH, 1991.)

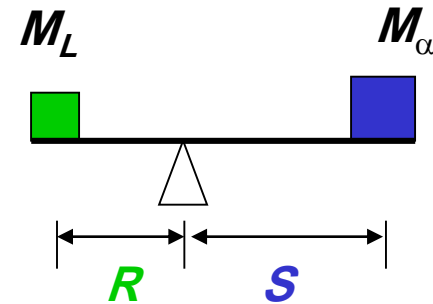
# The Lever Rule

- Tie line – connects the phases in equilibrium with each other - essentially an isotherm



How much of each phase?

Think of it as a lever (teeter-totter)



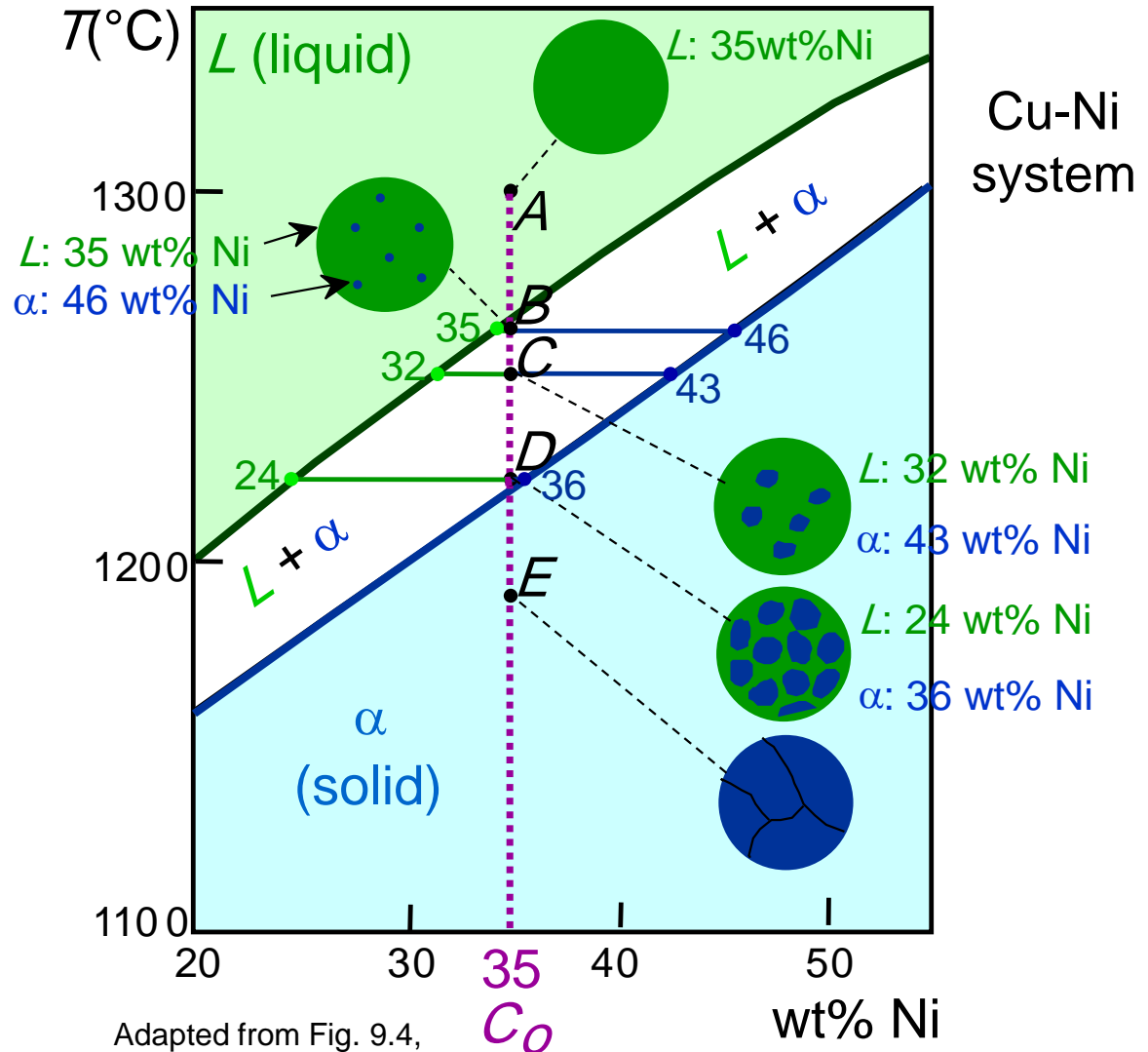
$$M_{\alpha} \cdot S = M_L \cdot R$$

$$W_L = \frac{M_L}{M_L + M_{\alpha}} = \frac{S}{R + S} = \frac{C_{\alpha} - C_0}{C_{\alpha} - C_L}$$

$$W_{\alpha} = \frac{R}{R + S} = \frac{C_0 - C_L}{C_{\alpha} - C_L}$$

# Ex: Cooling in a Cu-Ni Binary

- Phase diagram: Cu-Ni system.
- System is:
  - binary  
i.e., 2 components: Cu and Ni.
  - isomorphous  
i.e., complete solubility of one component in another;  $\alpha$  phase field extends from 0 to 100 wt% Ni.
- Consider  $C_0 = 35 \text{ wt\%Ni}$ .



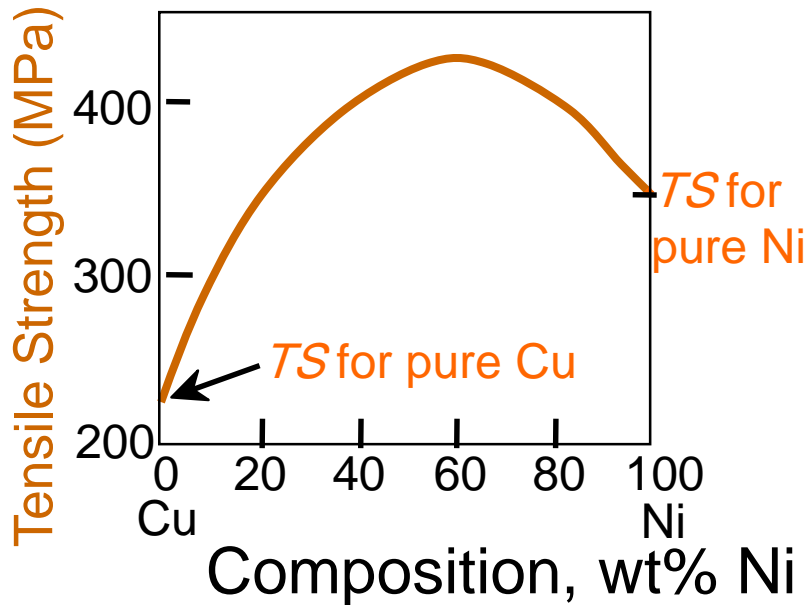
Adapted from Fig. 9.4,  
Callister 7e.



# Mechanical Properties: Cu-Ni System

- Effect of solid solution strengthening on:

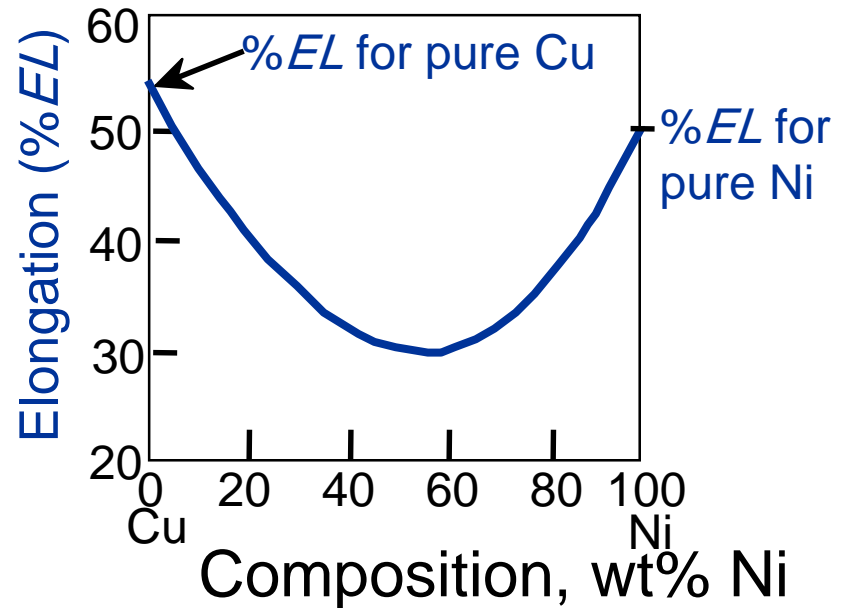
--Tensile strength ( $TS$ )



Adapted from Fig. 9.6(a), Callister 7e.

--Peak as a function of  $C_0$

--Ductility ( $\%EL, \%AR$ )



Adapted from Fig. 9.6(b), Callister 7e.

--Min. as a function of  $C_0$

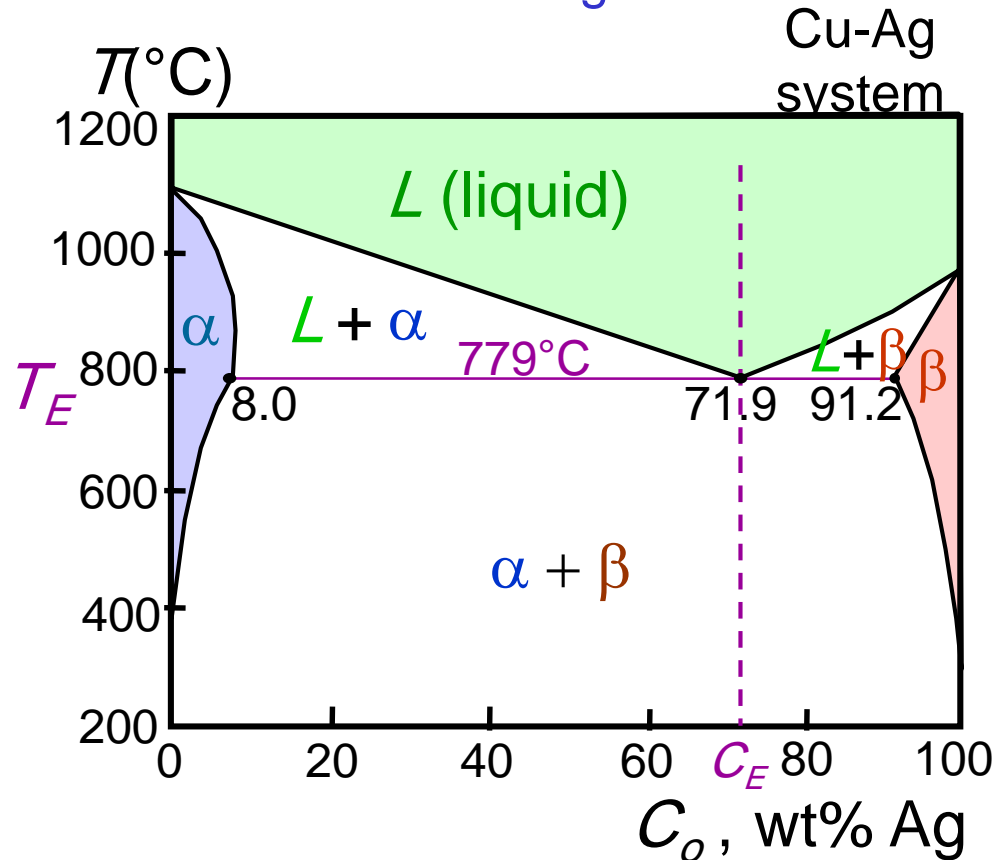
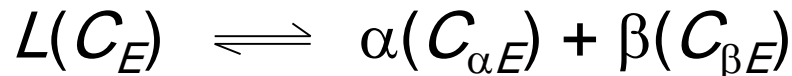
# Binary-Eutectic Systems

2 components

has a special composition with a min. melting T.

Ex.: Cu-Ag system

- 3 single phase regions ( $L$ ,  $\alpha$ ,  $\beta$ )
- Limited solubility:  
 $\alpha$ : mostly Cu  
 $\beta$ : mostly Ag
- $T_E$ : No liquid below  $T_E$
- $C_E$ : Min. melting  $T_E$  composition
- **Eutectic transition**



Adapted from Fig. 9.7,  
Callister 7e.

# EX: Pb-Sn Eutectic System (1)

- For a 40 wt% Sn-60 wt% Pb alloy at 150°C, find...

--the phases present:  $\alpha + \beta$

--compositions of phases:

$$C_o = 40 \text{ wt\% Sn}$$

$$C_\alpha = 11 \text{ wt\% Sn}$$

$$C_\beta = 99 \text{ wt\% Sn}$$

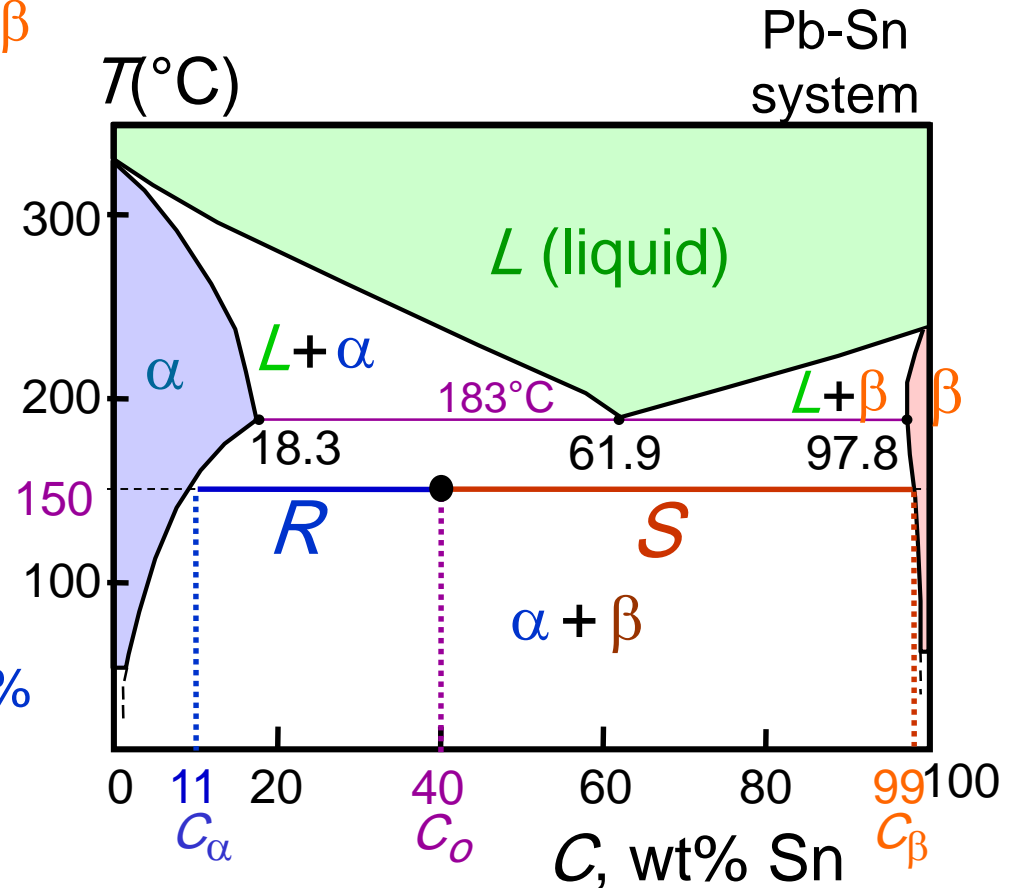
--the relative amount of each phase:

$$W_\alpha = \frac{S}{R+S} = \frac{C_\beta - C_o}{C_\beta - C_\alpha}$$

$$= \frac{99 - 40}{99 - 11} = \frac{59}{88} = 67 \text{ wt\%}$$

$$W_\beta = \frac{R}{R+S} = \frac{C_o - C_\alpha}{C_\beta - C_\alpha}$$

$$= \frac{40 - 11}{99 - 11} = \frac{29}{88} = 33 \text{ wt\%}$$



Adapted from Fig. 9.8,  
Callister 7e.

# EX: Pb-Sn Eutectic System (2)

- For a 40 wt% Sn-60 wt% Pb alloy at 200°C, find...

--the phases present:  $\alpha + L$

--compositions of phases:

$$C_O = 40 \text{ wt\% Sn}$$

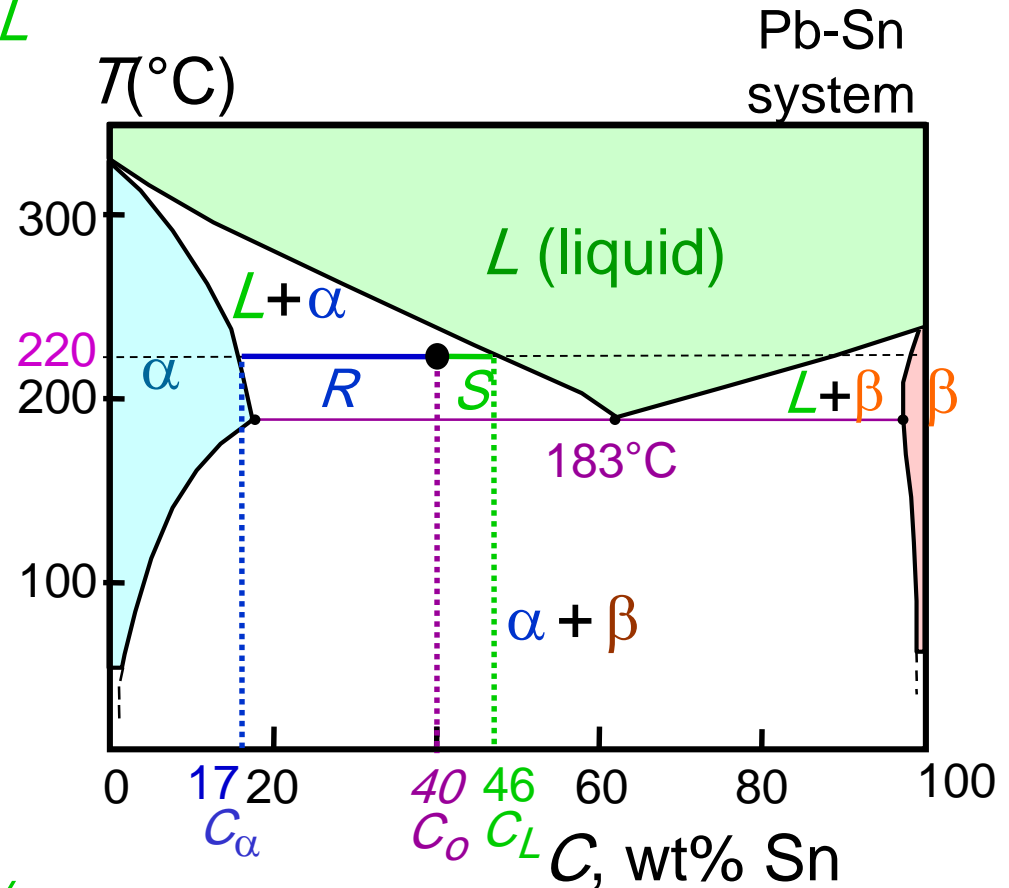
$$C_\alpha = 17 \text{ wt\% Sn}$$

$$C_L = 46 \text{ wt\% Sn}$$

--the relative amount of each phase:

$$W_\alpha = \frac{C_L - C_O}{C_L - C_\alpha} = \frac{46 - 40}{46 - 17} = \frac{6}{29} = 21 \text{ wt\%}$$

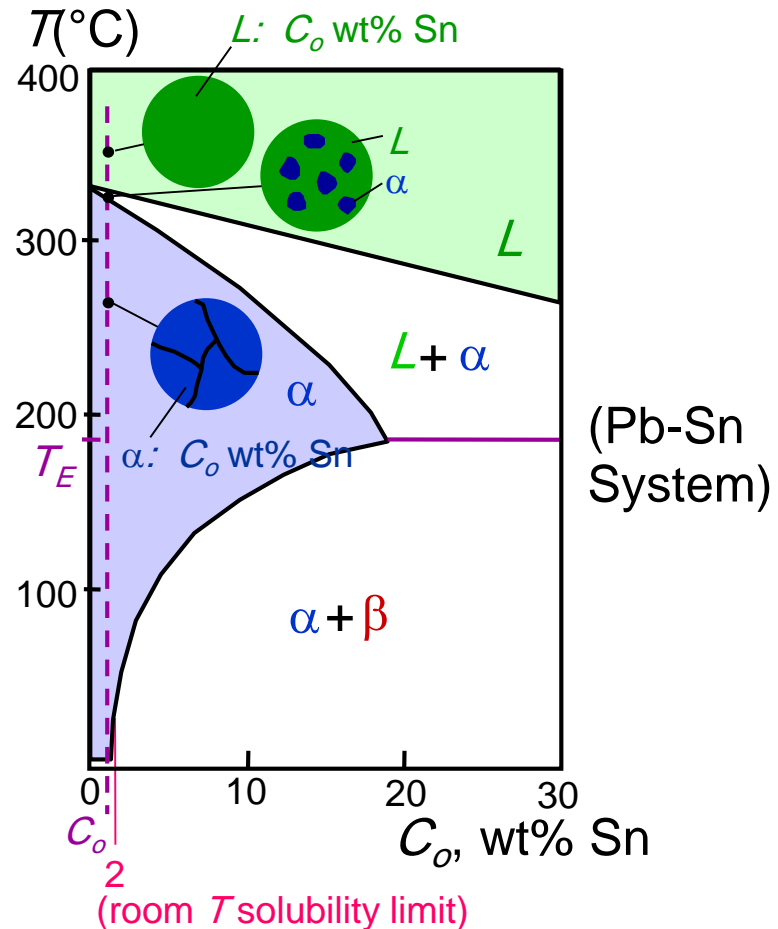
$$W_L = \frac{C_O - C_\alpha}{C_L - C_\alpha} = \frac{23}{29} = 79 \text{ wt\%}$$



Adapted from Fig. 9.8, Callister 7e.

# Microstructures in Eutectic Systems: I

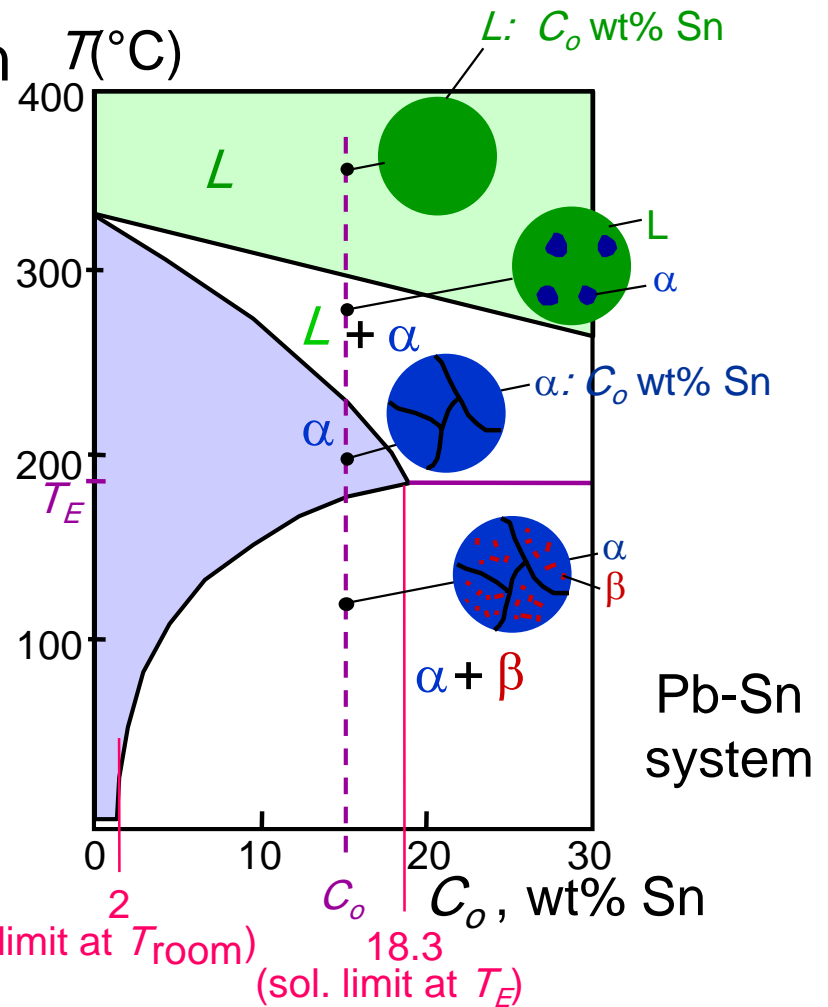
- $C_o < 2 \text{ wt\% Sn}$
- Result:
  - at extreme ends
  - polycrystal of  $\alpha$  grains  
i.e., only one solid phase.



Adapted from Fig. 9.11,  
Callister 7e.

# Microstructures in Eutectic Systems: II

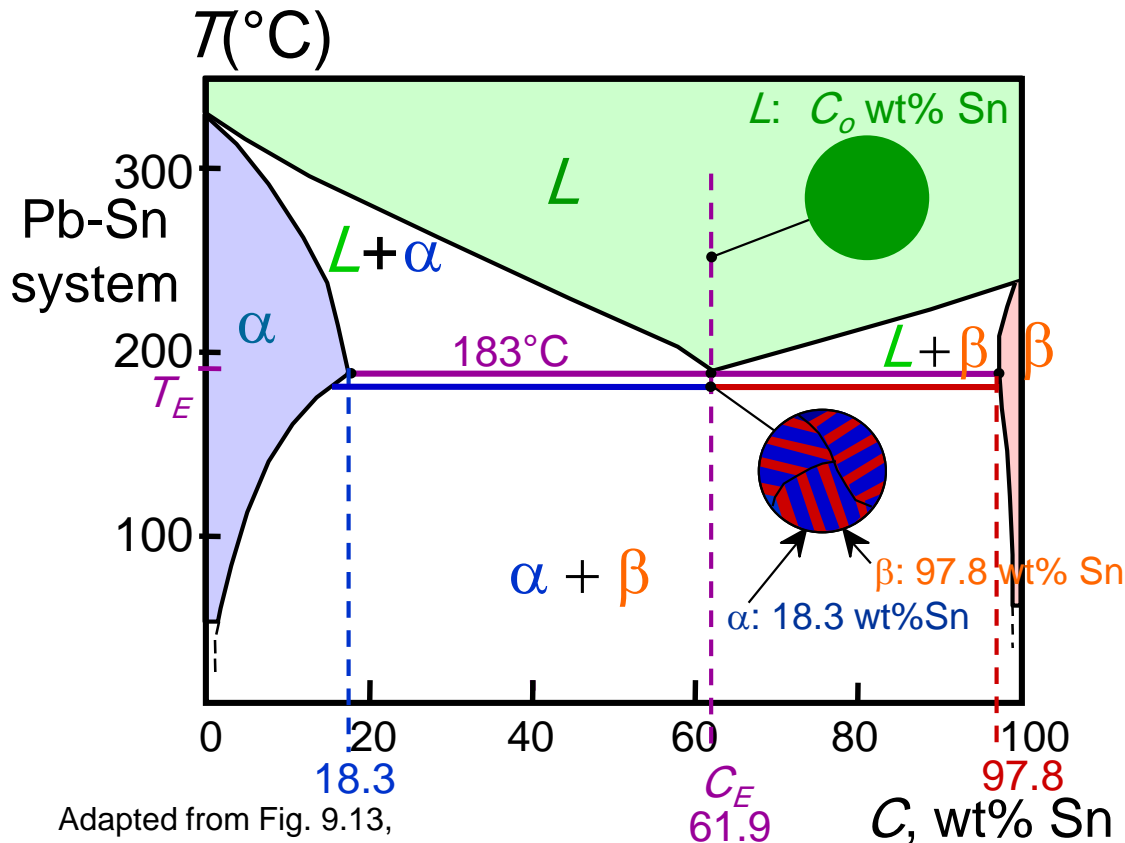
- $2 \text{ wt\% Sn} < C_o < 18.3 \text{ wt\% Sn}$
- Result:
  - Initially liquid +  $\alpha$
  - then  $\alpha$  alone
  - finally two phases
    - $\alpha$  polycrystal
    - fine  $\beta$ -phase inclusions



Adapted from Fig. 9.12,  
Callister 7e.

# Microstructures in Eutectic Systems: II

- $C_0 = C_E$
- Result: Eutectic microstructure (lamellar structure)  
--alternating layers (lamellae) of  $\alpha$  and  $\beta$  crystals.



Adapted from Fig. 9.13,  
*Callister 7e.*

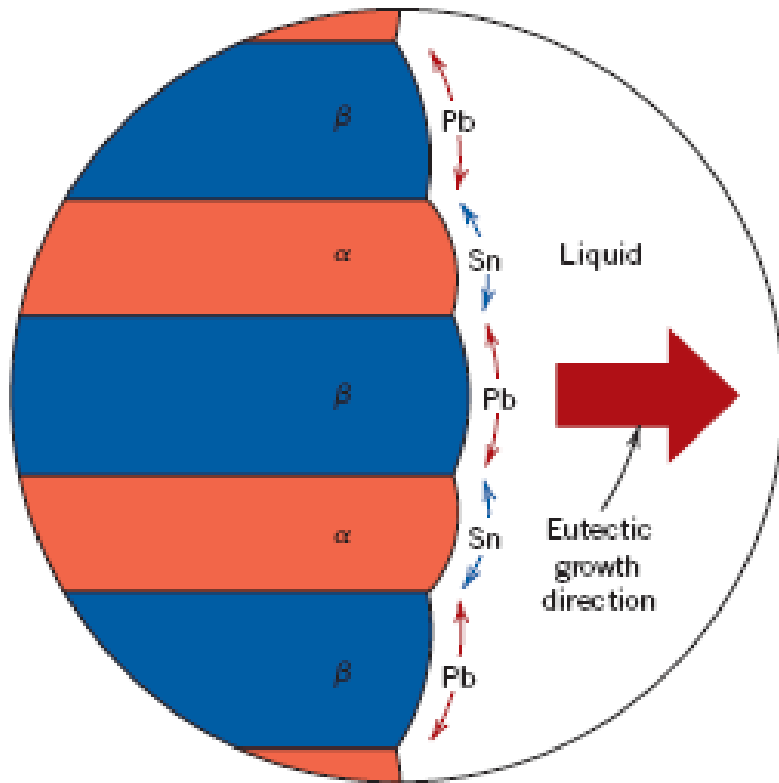
Micrograph of Pb-Sn  
eutectic  
microstructure



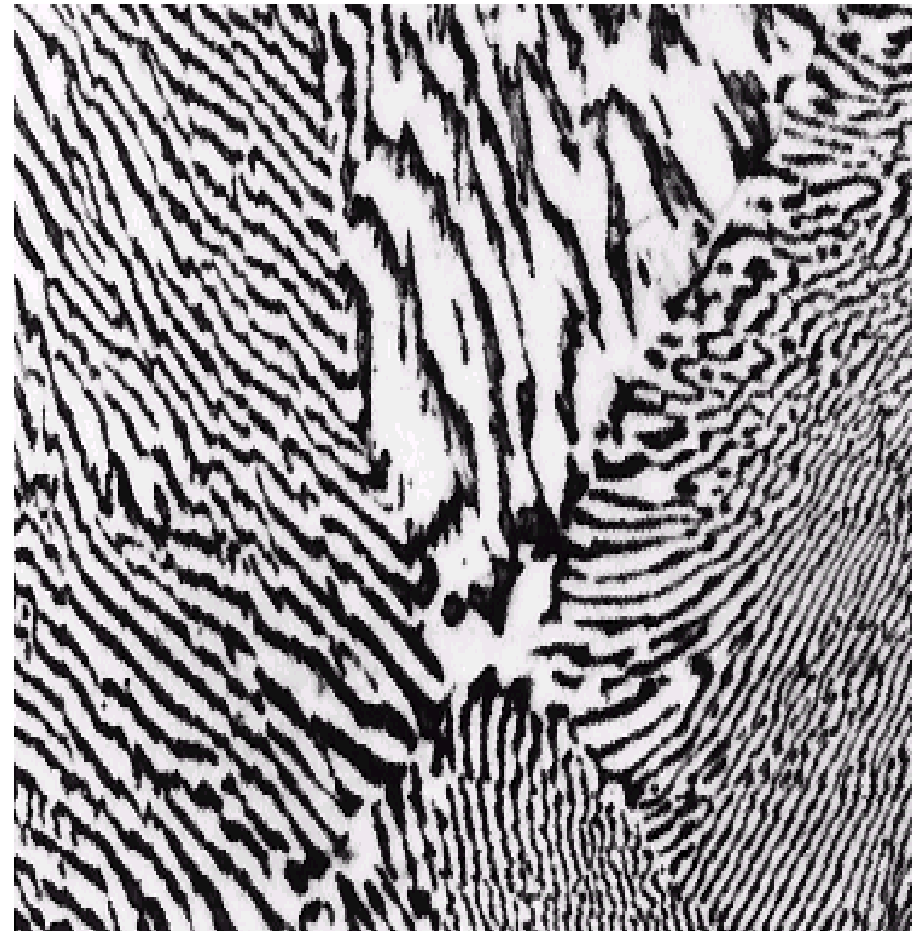
160  $\mu\text{m}$

Adapted from Fig. 9.14, *Callister 7e.*

# Lamellar Eutectic Structure



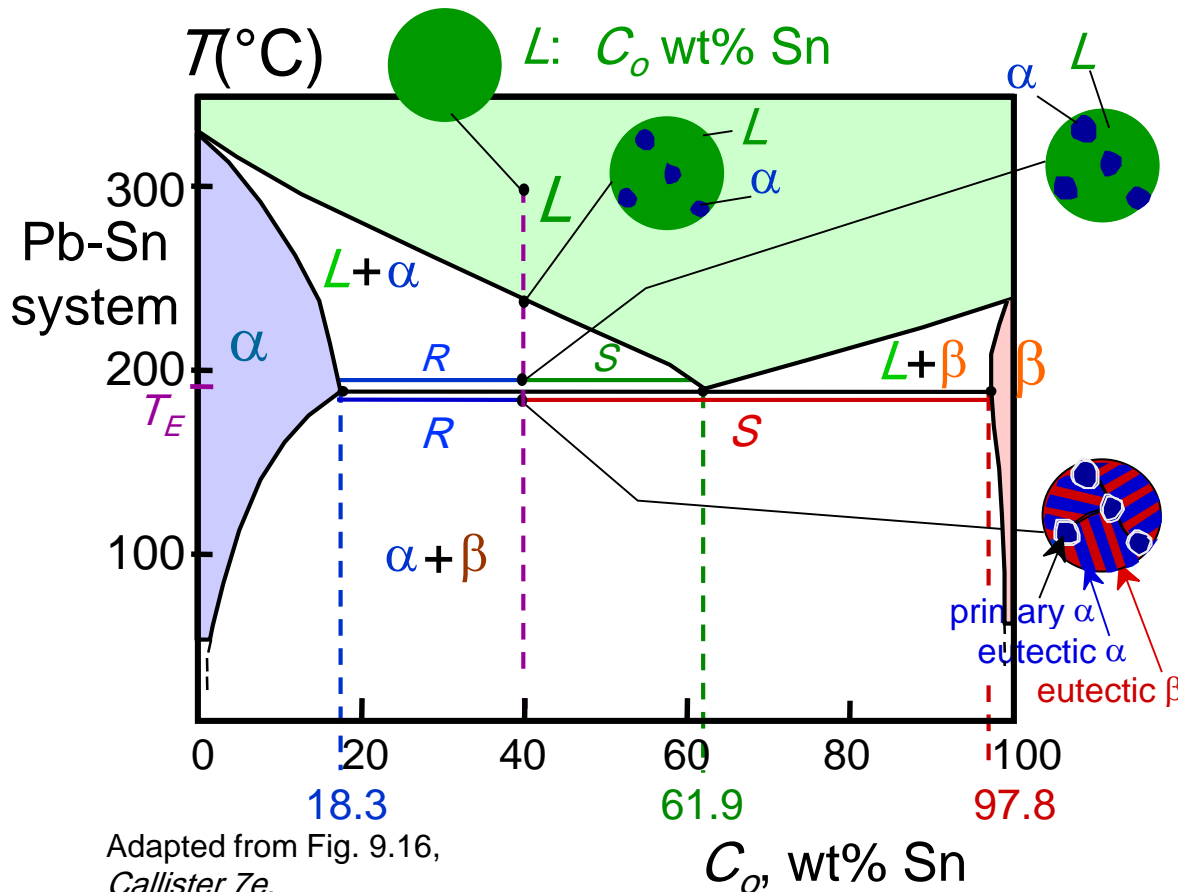
Adapted from Figs. 9.14 & 9.15, *Callister 7e*.





# Microstructures in Eutectic Systems: IV

- 18.3 wt% Sn <  $C_o$  < 61.9 wt% Sn
- Result:  $\alpha$  crystals and a eutectic microstructure

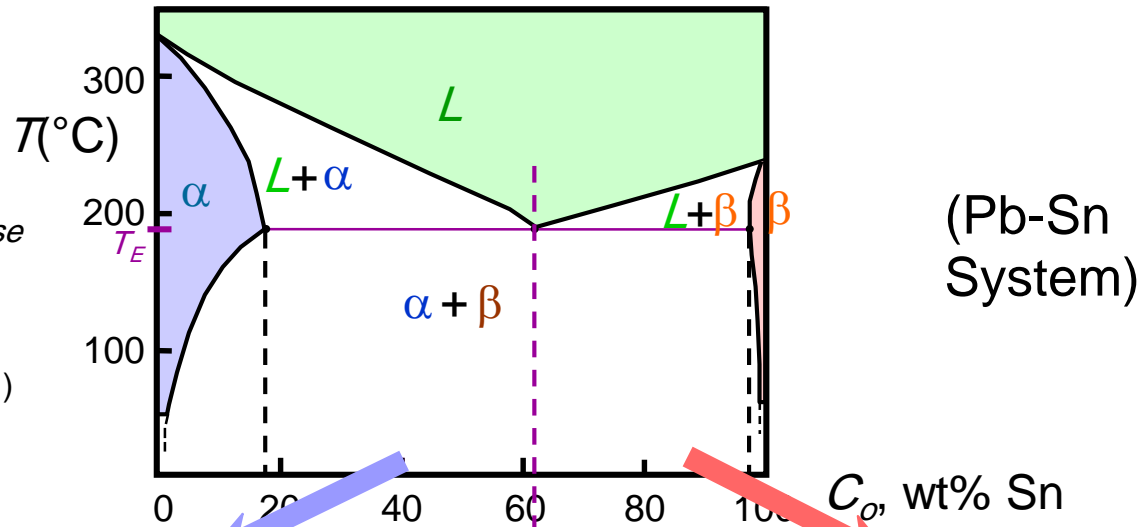


Adapted from Fig. 9.16,  
Callister 7e.

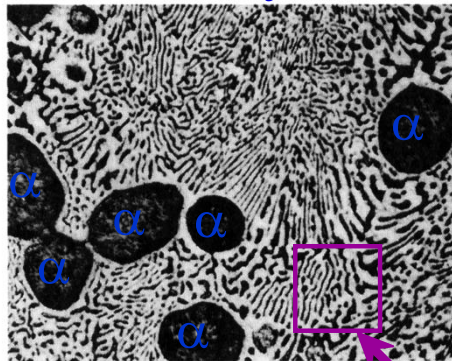
- Just above  $T_E$ :  
 $C_{\alpha} = 18.3$  wt% Sn  
 $C_L = 61.9$  wt% Sn  
 $W_{\alpha} = \frac{S}{R+S} = 50$  wt%  
 $W_L = (1 - W_{\alpha}) = 50$  wt%
- Just below  $T_E$ :  
 $C_{\alpha} = 18.3$  wt% Sn  
 $C_{\beta} = 97.8$  wt% Sn  
 $W_{\alpha} = \frac{S}{R+S} = 73$  wt%  
 $W_{\beta} = 27$  wt%

# Hypoeutectic & Hypereutectic

Adapted from Fig. 9.8, *Callister 7e*. (Fig. 9.8 adapted from *Binary Phase Diagrams*, 2nd ed., Vol. 3, T.B. Massalski (Editor-in-Chief), ASM International, Materials Park, OH, 1990.)



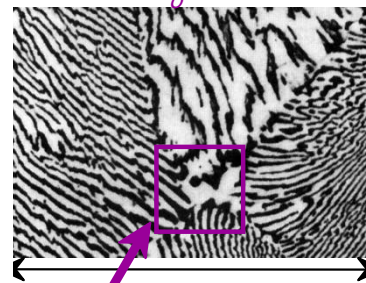
hypoeutectic:  $C_0 = 50 \text{ wt\% Sn}$



Adapted from Fig. 9.17, *Callister 7e*.

eutectic 61.9

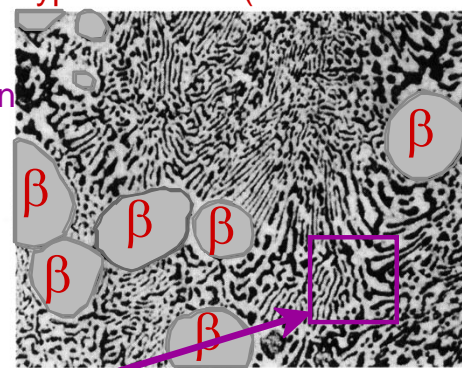
eutectic:  $C_0 = 61.9 \text{ wt\% Sn}$



eutectic micro-constituent

Adapted from Fig. 9.14, *Callister 7e*.

hypereutectic: (illustration only)



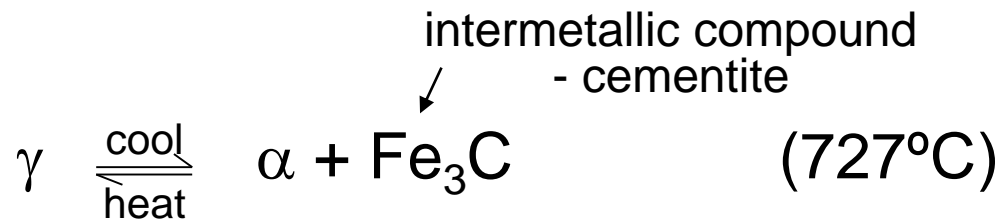
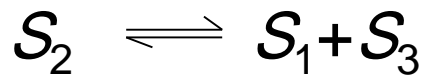
Adapted from Fig. 9.17, *Callister 7e*. (Illustration only)

# Eutectoid

- **Eutectic** - liquid in equilibrium with two solids



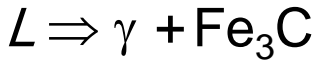
- **Eutectoid** - solid phase in equilibrium with two solid phases



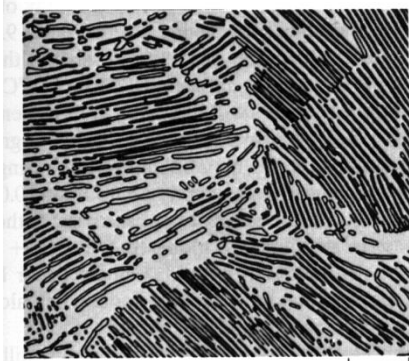
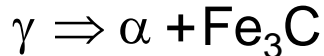
# Iron-Carbon (Fe-C) Phase Diagram

- 2 important points

-Eutectic (A):



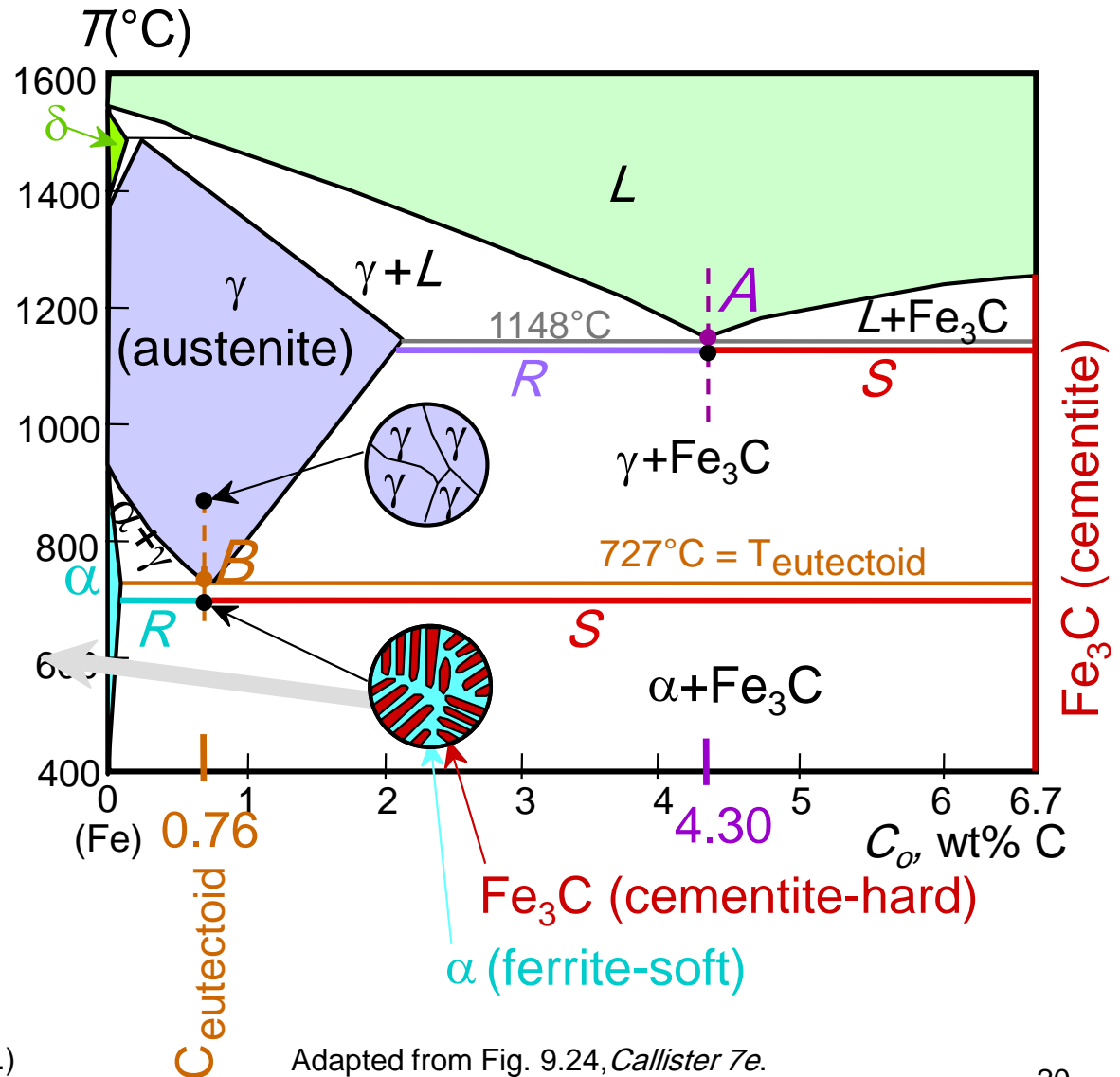
-Eutectoid (B):



120  $\mu\text{m}$

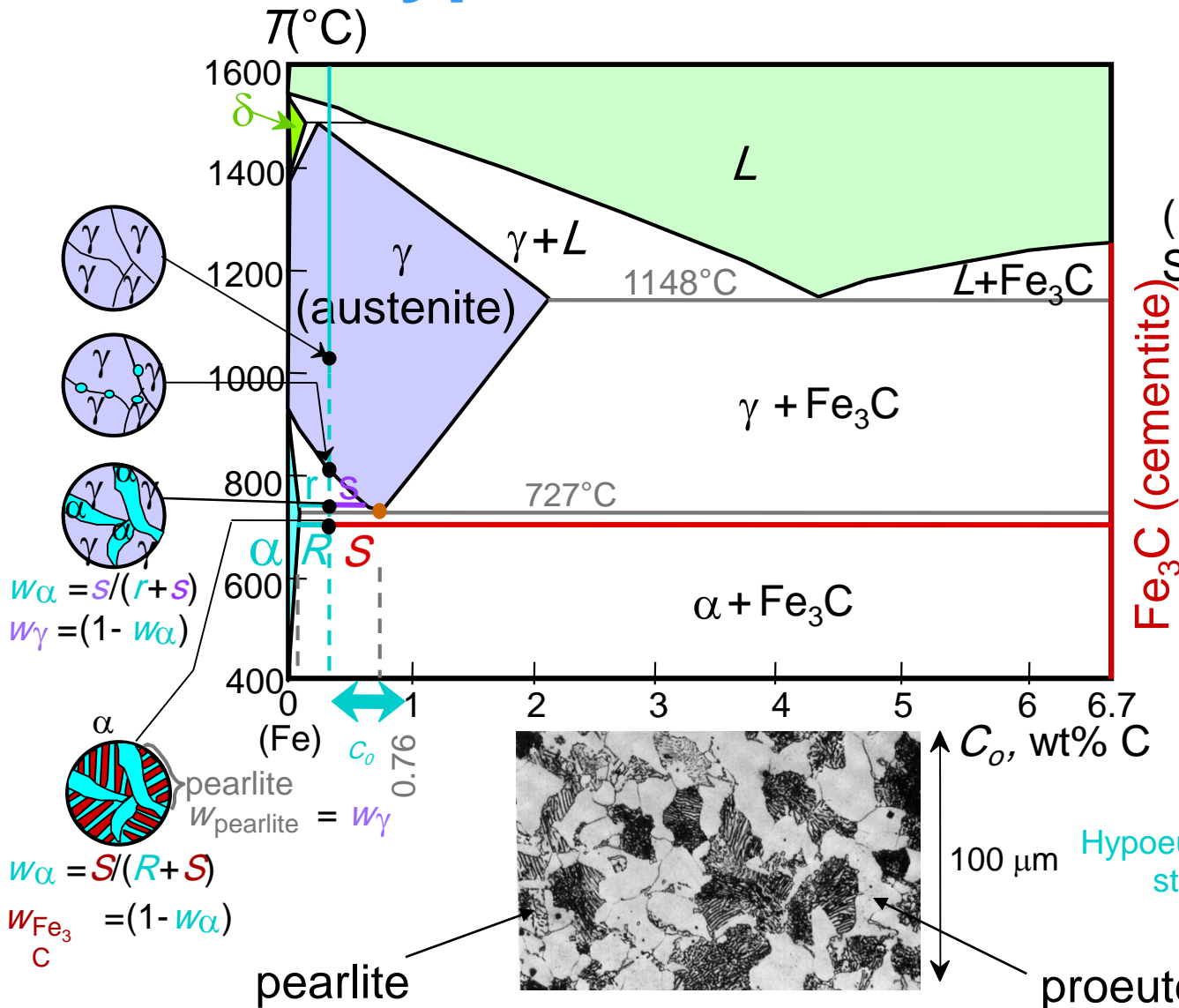
Result: Pearlite = alternating layers of  $\alpha$  and  $\text{Fe}_3\text{C}$  phases

(Adapted from Fig. 9.27, Callister 7e.)



Adapted from Fig. 9.24, Callister 7e.

# Hypoeutectoid Steel



(Fe-C System)

Adapted from Figs. 9.24 and 9.29, *Callister 7e*. (Fig. 9.24 adapted from *Binary Alloy Phase Diagrams*, 2nd ed., Vol. 1, T.B. Massalski (Ed.-in-Chief), ASM International, Materials Park, OH, 1990.)

Adapted from Fig. 9.30, *Callister 7e*.



# Example: Phase Equilibria

For a 99.6 wt% Fe-0.40 wt% C at a temperature just below the eutectoid, determine the following

- a) composition of  $\text{Fe}_3\text{C}$  and ferrite ( $\alpha$ )
- b) the amount of carbide (cementite) in grams that forms per 100 g of steel
- c) the amount of pearlite and proeutectoid ferrite ( $\alpha$ )

# Phase Equilibria

**Solution:** a) composition of Fe<sub>3</sub>C and ferrite (α)

b) the amount of carbide (cementite) in grams that forms per 100 g of steel

$$C_o = 0.40 \text{ wt\% C}$$

$$C_\alpha = 0.022 \text{ wt\% C}$$

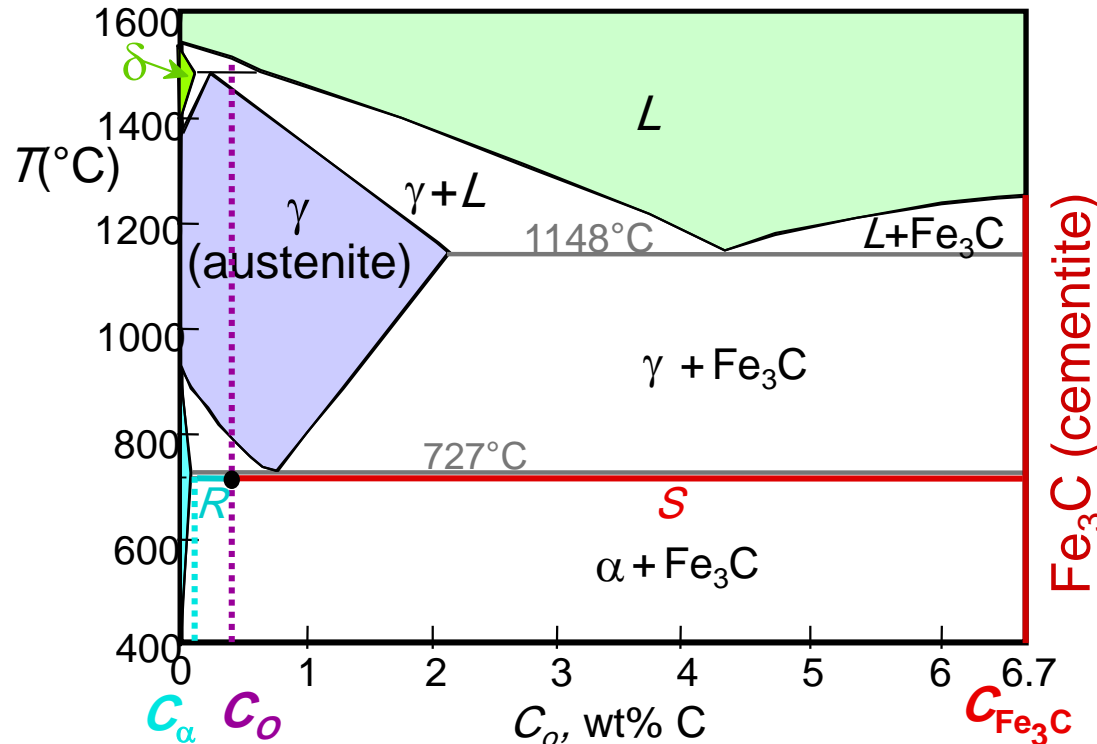
$$C_{Fe_3C} = 6.70 \text{ wt\% C}$$

$$\frac{Fe_3C}{Fe_3C + \alpha} = \frac{C_o - C_\alpha}{C_{Fe_3C} - C_\alpha} \times 100$$

$$= \frac{0.4 - 0.022}{6.7 - 0.022} \times 100 = 5.7g$$

$$Fe_3C = 5.7 \text{ g}$$

$$\alpha = 94.3 \text{ g}$$





# Chapter 9 – Phase Equilibria

c. the amount of pearlite and proeutectoid ferrite ( $\alpha$ )

note: amount of pearlite = amount of  $\gamma$  just above  $T_E$

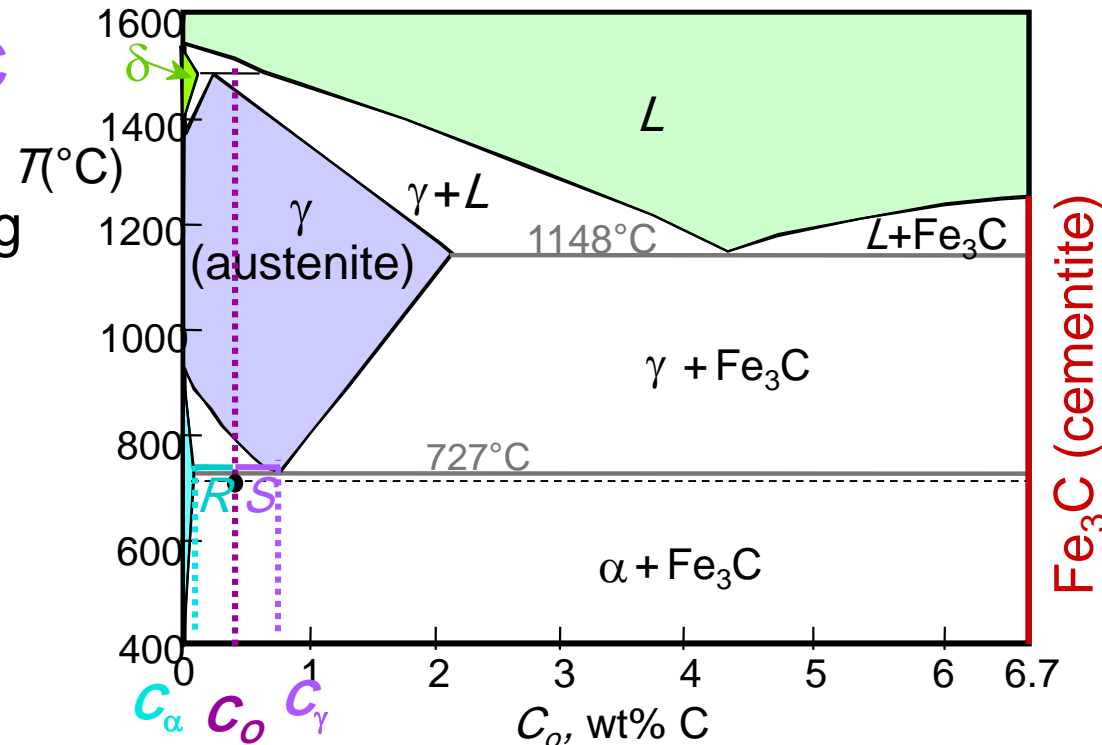
$$C_o = 0.40 \text{ wt\% C}$$

$$C_\alpha = 0.022 \text{ wt\% C}$$

$$C_{\text{pearlite}} = C_\gamma = 0.76 \text{ wt\% C}$$

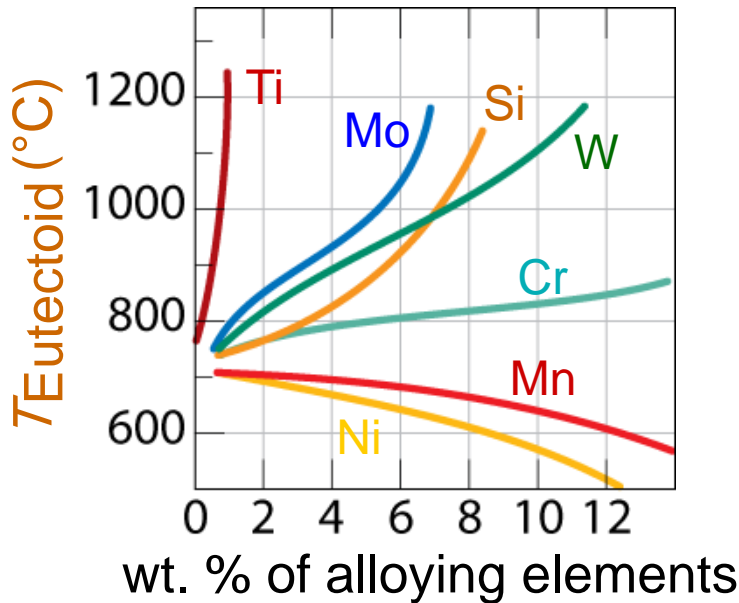
$$\frac{\gamma}{\gamma + \alpha} = \frac{C_o - C_\alpha}{C_\gamma - C_\alpha} \times 100 = 51.2 \text{ g}$$

pearlite = 51.2 g  
proeutectoid  $\alpha$  = 48.8 g



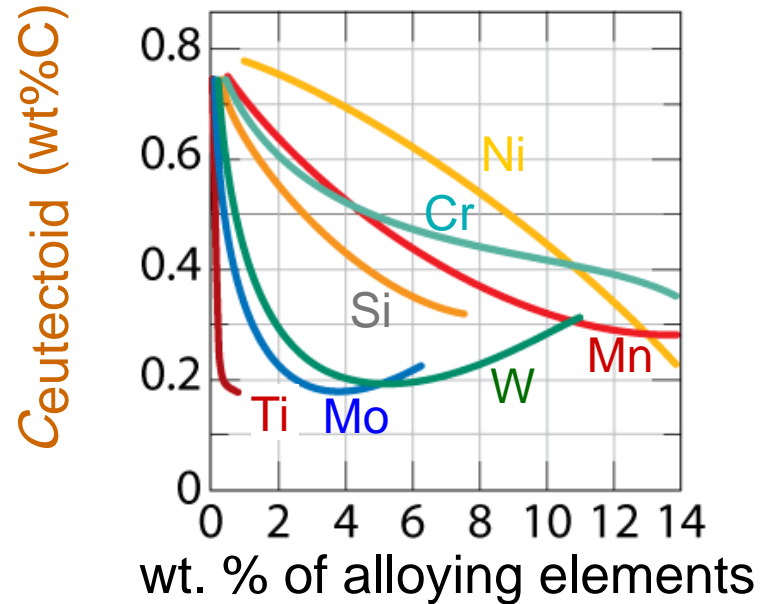
# Alloying Steel with More Elements

- $T_{\text{eutectoid}}$  changes:



Adapted from Fig. 9.34, *Callister 7e*. (Fig. 9.34 from Edgar C. Bain, *Functions of the Alloying Elements in Steel*, American Society for Metals, 1939, p. 127.)

- $C_{\text{eutectoid}}$  changes:



Adapted from Fig. 9.35, *Callister 7e*. (Fig. 9.35 from Edgar C. Bain, *Functions of the Alloying Elements in Steel*, American Society for Metals, 1939, p. 127.)

# Summary

- **Phase diagrams** are useful tools to determine:
  - the number and types of phases,
  - the wt% of each phase,
  - and the **composition** of each phasefor a given  $T$  and composition of the system.
- Alloying to produce a solid solution usually
  - increases the tensile strength ( $TS$ )
  - decreases the ductility.
- Binary **eutectics** and binary **eutectoids** allow for a range of microstructures.