Entropy

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ENTROPY

- Entropy and the Clausius Inequality

➢ The second law of thermodynamics leads to the definition of a new property called entropy, a quantitative measure of microscopic disorder for a system

➢ Entropy is a measure of energy that is no longer available to perform useful work within the current environment

➢ The definition of entropy is based on the Clausius inequality given by

$$\oint \frac{\delta Q}{T} \leq 0$$

- It is designated ***S*** and is defined as

$$dS = \left(\frac{\delta Q}{T} \right)_{\text{int}\, rev}$$

- Entropy - extensive properties of a system

- Sometimes referred as total entropy **(kJ/K)**

- Entropy per unit mass is an intensive properties and has the unit **(kJ/kg.K)**

- Entropy change of a system during a process :

$$\boxed{\Delta S = S_2 - S_1 = \int_1^2 \left(\frac{\delta Q}{T} \right)_{\text{int}\, rev} \qquad (kJ / K)}$$

- During the process, entropy change of a closed system during an irreversible process is always greater than the entropy transfer
- Some entropy generated or created during an irreversibility process, this generation is due to the presence of irreversibility.
- The entropy generated during a process is called entropy generation (***S_{gen}***)
- Entropy change can be rewritten as

$$\boxed{\Delta S = S_2 - S_1 = \int_1^2 \left(\frac{\partial Q}{T}\right) + S_{gen} \qquad (kJ/K)}$$

***S_{gen}* = 0** (No entropy generated during reversible process)

- For an isolated system (or simply an adiabatic closed system) the heat transfer is zero, so

$$\Delta S_{isolated} \geq 0$$

- Entropy of the process also included system and surrounding of the system

$$S_{gen} = \Delta S_{total} = \Delta S_{sys} + \Delta S_{surrounding} \geq 0$$

- The increase of entropy principle can be summarized as follows

$$S_{gen} \begin{cases} > & \quad irreversible\ \ process \\ = \ \ 0 & \ reversible\ \ process \\ < & \quad impossible\ \ process \end{cases}$$

Example (1)

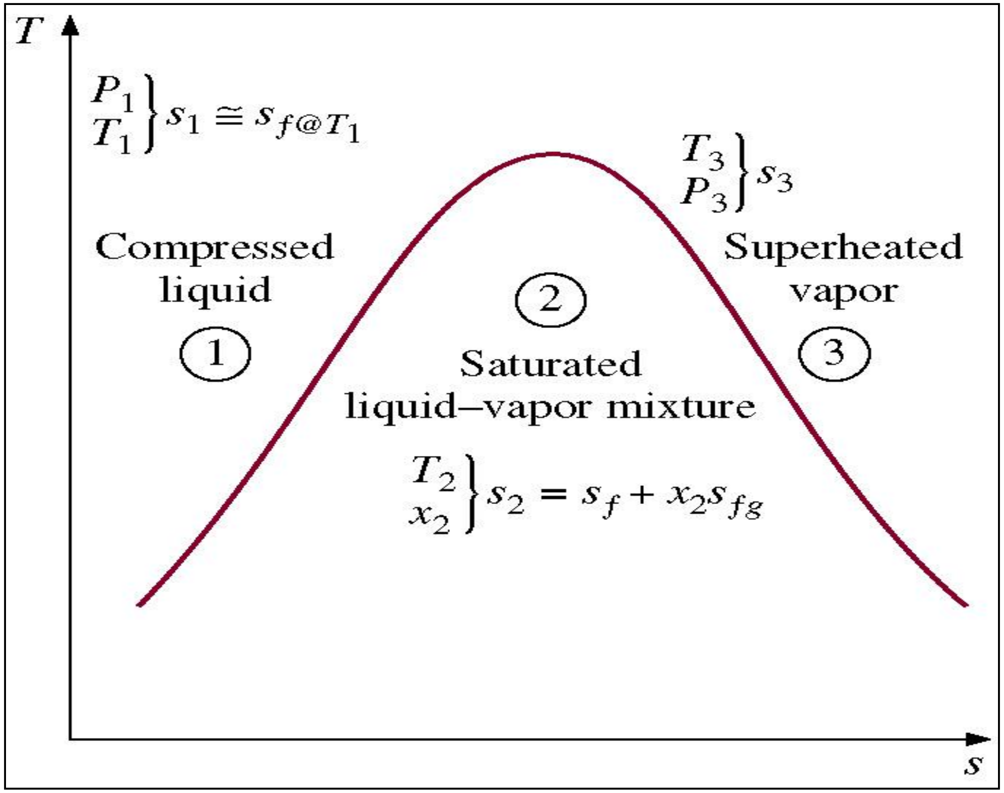
A heat source at 800 K loses 2000 kJ of heat to a sink at (*a*) 500 K and (*b*) 750 K. Determine which heat transfer process is more irreversible.

1)Entropy Change of Pure Substance

- The value of entropy at a specified state is determined just like any other property.

Example (2)

A piston–cylinder device initially contains 1.5 kg of liquid water at 150 KPa and 20 C. The water is now heated at constant pressure by the addition of 4000 KJ of heat. Determine the entropy change of the water during this process.



- **Entropy Change of Pure Substance**

  - In the compressed liquid & superheated vapor regions , it can be obtained directly from tables at the specific state.

  - In the saturated mixture region, it is determined by using quality, *x*

  - In the absence of compressed liquid data, the entropy can be approximated by the entropy of the saturated liquid at the given temperature

$$s = s_f + x s_{fg} \qquad (kJ/kg.K)$$

$$s_{@T,P} \cong s_{f@T} \qquad compressed\ liquid \quad (kJ/kg.K)$$

$$entropy\ change\ of\ a\ specified\ mass$$

$$\Delta S = m \Delta s = m(s_2 - s_1) \qquad (kJ/K)$$

Isentropic Process

- Entropy of the fixed mass can be changed by ***heat transfer*** and ***irreversibility***

- Entropy of a fixed mass will not change during a process that is ***internally reversible*** and ***adiabatic***

- Process during which the entropy remains constant is called ***isentropic process*** $(\Delta s = 0 \quad or \quad s_2 = s_1)$

- Isentropic process enable to **define efficiencies** for process to compare the actual performance of the device to the performance under idealized condition.

- ***Isentropic process***
  - \# internally reversible process
  - \# adiabatic process

Example (3)

Steam enters an adiabatic turbine at 5 MPa and 450 C and leaves at a pressure of 1.4 MPa. Determine the work output of the turbine per unit mass of steam if the process is reversible..

Entropy Relations

$$generally \quad \delta Q - \delta W = dU$$

$$where \quad dS = \frac{\delta Q}{T} \qquad ; \; \delta W = PdV$$

$$replaced ....$$

$$TdS - PdV = dU$$

$$TdS = dU + PdV \qquad (kJ)$$

- relations per unit mass

$$\boxed{Tds = du + Pdv \quad \left(kJ/kg\right)}\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

as we know....

$$h = u + Pv \quad (differentiate)$$

$$dh = du + Pdv + vdP \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

$$Tds = dh - \cancel{Pdv} - vdP + \cancel{Pdv} \ldots\ldots\ldots\ldots(2)\, in\, (1)$$

$$\boxed{Tds = dh - vdP \quad (kJ/kg)}$$

also re-write as

$$\boxed{\begin{aligned} ds &= \frac{du}{T} + P\frac{dv}{T} \\ ds &= \frac{dh}{T} - v\frac{dP}{T} \end{aligned}}$$

2) Entropy Change of Liquids and Solids (incompressible subs)

- Liquids & solids can be approximated as incompressible substance (since specific volume nearly remains constant during a process)
- $dv = 0$

$$ds = \frac{du}{T} = C\frac{dT}{T}$$

$$where\ C_p = C_v = C\ (\,for\ incompressible\ \ subs\tan ce)$$

$$entropy\ \ change\ \ during\ \ a\ \ process:$$

$$liquids, solids \quad s_2 - s_1 = \int_1^2 C(T)\frac{dT}{T}$$

$$\boxed{s_2 - s_1 = C_{av}\ln\left(\frac{T_2}{T_1}\right)} \qquad (kJ/kg.K)$$

$$where\ \ C_{av}\, is\ \ the\ \ average\ \ specific\ heat\ \ of\ \ subs\tan ces$$

- A relation for isentropic process of liquids & solids is obtained by setting entropy change equal to zero

Isentropic process

$$\boxed{\begin{aligned} & s_2 - s_1 = 0 \\ & C_{av} \ln\left(\frac{T_2}{T_1}\right) = 0 \\ & \textit{thus,} \quad T_2 = T_1 \end{aligned}}$$

so : isentropic process of an incompressible substances is also isothermal

Example (4) Liquid methane is commonly used in various cryogenic applications. The critical temperature of methane is 191 K (or -82 C), and thus methane must be maintained below 191 K to keep it in liquid phase. The properties of liquid methane at various temperatures and pressures are given in Table 7–1. Determine the entropy change of liquid methane as it undergoes a process from 110 K and 1 MPa to 120 K and 5 MPa (*a*) using tabulated properties and (*b*) approximating liquid methane as an incompressible substance. What is the error involved in the latter case?

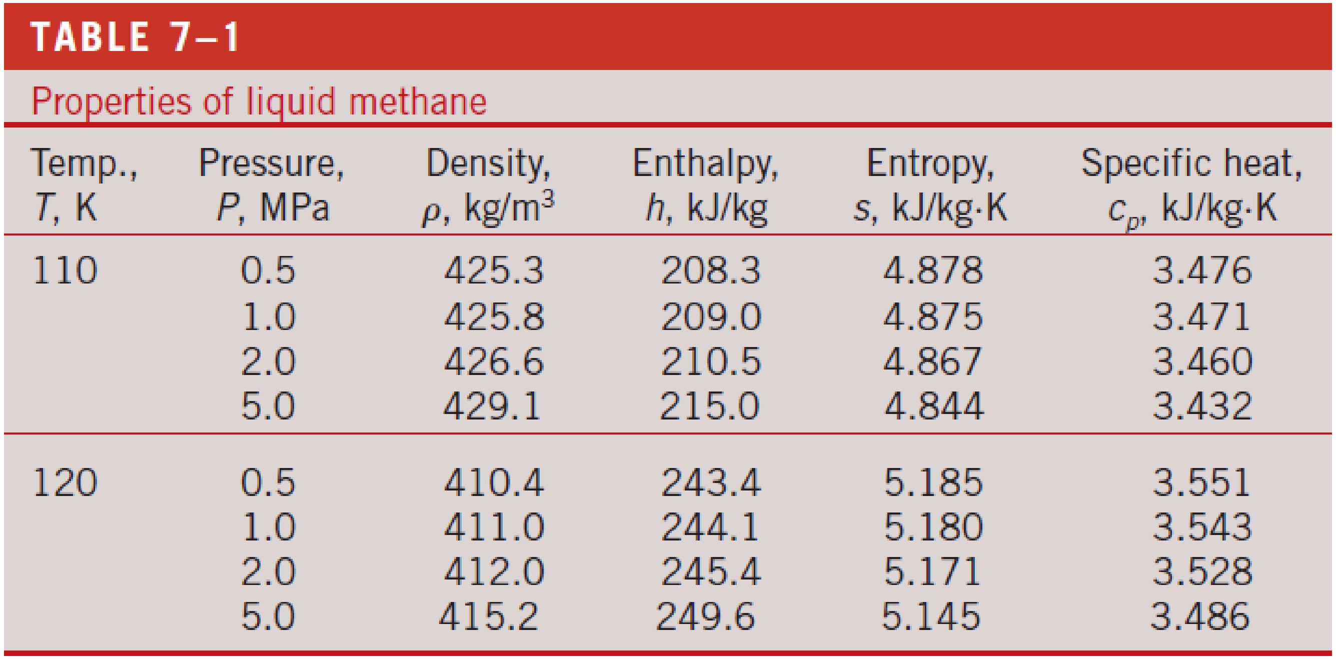


TABLE 7–1

Properties of liquid methane

Temp., T, K	Pressure, P, MPa	Density, ρ, kg/m^3	Enthalpy, h, kJ/kg	Entropy, s, kJ/kg·K	Specific heat, c_p, kJ/kg·K
110	0.5	425.3	208.3	4.878	3.476
	1.0	425.8	209.0	4.875	3.471
	2.0	426.6	210.5	4.867	3.460
	5.0	429.1	215.0	4.844	3.432
120	0.5	410.4	243.4	5.185	3.551
	1.0	411.0	244.1	5.180	3.543
	2.0	412.0	245.4	5.171	3.528
	5.0	415.2	249.6	5.145	3.486

3) Entropy Change of Ideal Gases

$$by\,u\sin g$$

$$du = C_v dT \qquad and \qquad dh = C_p dT$$

$$also \qquad Pv = RT \qquad to \quad get$$

$$P = \frac{RT}{v} \qquad and \qquad v = \frac{RT}{P}$$

By substituting du and P into the equation $ds = \frac{du}{T} + P\frac{dv}{T}$

We get

$$ds = C_v \frac{dT}{T} + P\frac{dv}{T} \qquad replaced \;\; P = \frac{RT}{v}$$

$$ds = C_v \frac{dT}{T} + R\frac{dv}{v} \qquad then \;\; \mathrm{int}\, egrated \;\; \int ds$$

$$s_2 - s_1 = \int_1^2 C_v \frac{dT}{T} + R\ln\left(\frac{v_2}{v_1}\right)$$

Also, by substituting dh and v into the equation
We get

$$ds = \frac{dh}{T} - v\frac{dP}{T}$$

$$ds = C_p \frac{dT}{T} - v\frac{dP}{T} \qquad replaced\ v = \frac{RT}{P}$$

$$ds = C_p \frac{dT}{T} - R\frac{dP}{P} \qquad then\ \ \text{int}\,egrated \int ds$$

$$s_2 - s_1 = \int_1^2 C_p \frac{dT}{T} - R\ln\left(\frac{P_2}{P_1}\right)$$

1) Constant Specific Heats (Approximately)

$$s_2 - s_1 = C_{v,av}\ln\left(\frac{T_2}{T_1}\right) + R\ln\left(\frac{v_2}{v_1}\right) \qquad (kJ/kg.K)$$

$$s_2 - s_1 = C_{p,av}\ln\left(\frac{T_2}{T_1}\right) - R\ln\left(\frac{P_2}{P_1}\right) \qquad (kJ/kg.K)$$

mole basis

$$\overline{s}_2 - \overline{s}_1 = \overline{C}_{v,av} \ln\left(\frac{T_2}{T_1}\right) + R_u \ln\left(\frac{v_2}{v_1}\right) \qquad (kJ/kmol.K)$$

$$\overline{s}_2 - \overline{s}_1 = \overline{C}_{p,av} \ln\left(\frac{T_2}{T_1}\right) - R_u \ln\left(\frac{P_2}{P_1}\right) \qquad (kJ/kmol.K)$$

2) Variable Specific Heats (Exactly)

- When the temperature change during a process is large and the specific heats of the ideal gas vary nonlinearly within the temperature range.
- Instead of performing these laborious integrals each time we have a new process, it is convenient to perform these integrals once and tabulate the results.
- For this purpose, we choose absolute zero as the reference temperature and define a function s° as

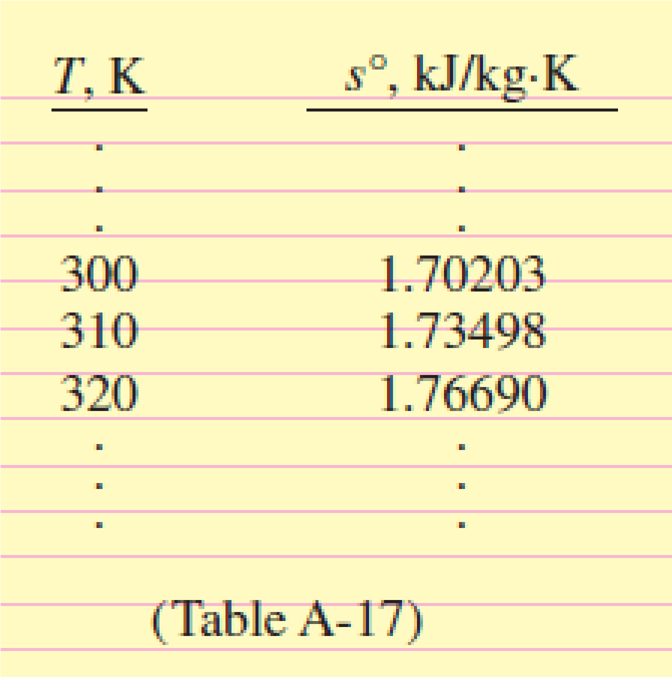
$$s^\circ = \int_0^T C_p(T)dT \quad or \quad C_v$$

- Obviously, s° is a function of temperature alone, and its value is zero at absolute zero temperature.

$$\int_1^2 C_p(T)dT = s_2^\circ - s_1^\circ$$

$$\therefore \quad s_2 - s_1 = s_2^\circ - s_1^\circ - R\ln\left(\frac{P_2}{P_1}\right)$$

$$\therefore \quad s_2 - s_1 = s_2^\circ - s_1^\circ + R\ln\left(\frac{v_2}{v_1}\right)$$



T, K	*s*°, kJ/kg·K
.	.
.	.
.	.
300	1.70203
310	1.73498
320	1.76690
.	.
.	.
.	.

(Table A-17)

Example (5) Air is compressed from an initial state of 100 kPa and 17C to a final state of 600 kPa and 57C. Determine the entropy change of air during this compression process by using (*a*) property values from the air table and (*b*) average specific heats.

Isentropic Process of Ideal Gases

1) Constant Specific Heats (Approximately) – isentropic relations

Isentropic process , s_2-s_1= 0

$$s_2 - s_1 = C_v \ln\left(\frac{T_2}{T_1}\right) + R\ln\left(\frac{v_2}{v_1}\right)$$

$$C_v \ln\left(\frac{T_2}{T_1}\right) = -R\ln\left(\frac{v_2}{v_1}\right)$$

$$\ln\left(\frac{T_2}{T_1}\right) = -\frac{R}{C_v}\ln\left(\frac{v_2}{v_1}\right)$$

$$\ln\left(\frac{T_2}{T_1}\right) = \ln\left(\frac{v_1}{v_2}\right)^{\frac{R}{C_v}}$$

where

$$R = C_p - C_v$$

$$k = {C_p}/{C_v}$$

- Isentropic relation for ideal gas for constant specific heats

$$\left(\frac{T_2}{T_1}\right) = \left(\frac{v_1}{v_2}\right)^{k-1}$$

$$\left(\frac{T_2}{T_1}\right) = \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}}$$

$$\left(\frac{P_2}{P_1}\right) = \left(\frac{v_1}{v_2}\right)^{k}$$

valid for

- *ideal gas*
- *isentropic process*
- *constant specific heats*

summarized relation

$$Tv^{(k-1)} = C$$

$$TP^{\left(\frac{1-k}{k}\right)} = C$$

$$Pv^{k} = C$$

2) Variable Specific Heats (Exact Analysis)

Isentropic process , s_2-s_1= o

$$s_2 - s_1 = 0$$

$$s_2 - s_1 = s_2^{o} - s_1^{o} - R\ln\left(\frac{P_2}{P_1}\right)$$

so, $$\boxed{s_2^{o} = s_1^{o} + R\ln\left(\frac{P_2}{P_1}\right)}$$

Relative pressure and relative specific volume

$$s_2^{\circ} = s_1^{\circ} + R\ln\left(\frac{P_2}{P_1}\right)$$

$$s_2^{\circ} - s_1^{\circ} = R\ln\left(\frac{P_2}{P_1}\right)$$

$$\frac{s_2^{\circ} - s_1^{\circ}}{R} = \ln\left(\frac{P_2}{P_1}\right)$$

relative pressure

$$\left(\frac{P_2}{P_1}\right) = \exp\left(\frac{s_2^{\circ} - s_1^{\circ}}{R}\right) = \frac{\exp\left(s_2^{\circ}/R\right)}{\exp\left(s_1^{\circ}/R\right)}$$

$$\boxed{\frac{P_2}{P_1} = \frac{P_{r2}}{P_{r1}} \qquad \text{dim}\,ensionless}$$

Ideal gas relation $$\frac{P_1 v_1}{T_1} = \frac{P_2 v_2}{T_2}$$

$$\frac{v_2}{v_1} = \frac{P_1 T_2}{T_1 P_2} = \frac{T_2}{T_1}\frac{P_1}{P_2}$$

$$= \frac{T_2}{T_1}\frac{P_{r1}}{P_{r2}}$$

relative specific volume

$$\left(\frac{v_2}{v_1}\right) == \frac{\left(T_2 / P_{r2}\right)}{\left(T_1 / P_{r1}\right)}$$

$$\boxed{\frac{v_2}{v_1} = \frac{v_{r2}}{v_{r1}} \qquad \dim ensionless}$$

Example (6)

Air is compressed in a car engine from 22 C and 95 kPa in a reversible and adiabatic manner. If the compression ratio $V1/V2$ of this engine is 8, determine the final temperature of the air.

Example (7)

Helium gas is compressed by an adiabatic compressor from an initial state of 100 KPa and 10 C to a final temperature of 160 C in a reversible manner. Determine the exit pressure of helium.