ENERGY ANALYSIS OF CLOSED SYSTEMS

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Processes and Cycles

- The prefix *iso-* is often used to designate a process for which a particular property remains constant.

- **Isothermal process,** for example, is a process during which the temperature *T* remains constant.
- **Isobaric process** is a process during which the pressure *P* remains constant;
- **Isochoric** (or **isometric**) **process** is a process during which the specific volume *v* remains constant.

- A system is said to have undergone a **cycle** if it returns to its initial state at the end of the process. That is, for a cycle the initial and final states are identical.

Boundary Work for Isochoric Process

Example (1)

A rigid tank contains air at 500 kPa and 150°C. As a result of heat transfer to the surroundings, the temperature and pressure inside the tank drop to 65°C and 400 kPa, respectively. Determine the boundary work done during this process.

$$W_b = \int_1^2 P\,\cancelto{0}{dV} = 0$$

Boundary Work for Isobaric Process

Example (2)

A frictionless piston–cylinder device contains 5 kg of steam at 400 kPa and 200 C. Heat is now transferred to the steam until the temperature reaches 250 C. If the piston is not attached to a shaft and its mass is constant, determine the work done by the steam during this process.

Boundary Work for Isothermal Process

Example (3)

A piston–cylinder device initially contains 0.4 m^3 of air at 100 kPa and 80°C. The air is now compressed to 0.1 m^3 in such a way that the temperature inside the cylinder remains constant. Determine the work done during this process.

Polytropic Process

During actual expansion and compression processes of gases, pressure and volume are often related by $PV^n = C$, where n and C are constants. A process of this kind is called a **polytropic process.**

The pressure for a polytropic process can be expressed as:-

$$P = C\, V^{-n}$$

Substituting this relation into **Boundary Work** Eq. , we obtain

$$W_b = \int_1^2 P\,dV = \int_1^2 CV^{-n}\,dV = C\frac{V_2^{-n+1} - V_1^{-n+1}}{-n+1} = \frac{P_2V_2 - P_1V_1}{1-n} \qquad \textbf{(4–9)}$$

since $C = P_1V_1^n = P_2V_2^n$. For an ideal gas ($PV = mRT$), this equation can also be written as

$$W_b = \frac{mR(T_2 - T_1)}{1-n} \qquad n \neq 1 \qquad (\text{kJ}) \qquad \textbf{(4–10)}$$

For the special case of $n = 1$ the boundary work becomes

$$W_b = \int_1^2 P\,dV = \int_1^2 CV^{-1}\,dV = PV\ln\left(\frac{V_2}{V_1}\right)$$

For an ideal gas this result is equivalent to the isothermal process discussed in the previous example.

Example (4): A piston–cylinder device contains 0.05 m^3 of a gas initially at 200 kPa. At this state, a linear spring that has a spring constant of 150 kN/m is touching the piston but exerting no force on it. Now heat is transferred to the gas, causing the piston to rise and to compress the spring until the volume inside the cylinder doubles. If the cross-sectional area of the piston is 0.25 m^2, determine (*a*) the final pressure inside the cylinder, (*b*) the total work done by the gas, and (*c*) the fraction of this work done against the spring to compress it.

Energy Balance for Closed Systems

$$Q_{\text{net,in}} - W_{\text{net,out}} = \Delta E_{\text{system}}$$

or

$$Q - W = \Delta E$$

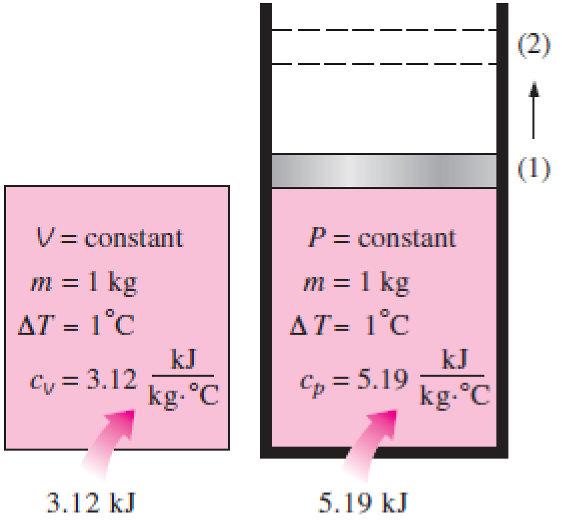
Example (5)

A piston–cylinder device contains 25 g of saturated water vapor that is maintained at a constant pressure of 300 kPa. A resistance heater within the cylinder is turned on and passes a current of 0.2 A for 5 min from a 120-V source. At the same time, a heat loss of 3.7 kJ occurs. (*a*) Show that for a closed system the boundary work W_b and the change in internal energy U in the first-law relation can be combined into one term, H, for a constantpressure process. (*b*) Determine the final temperature of the steam.

Specific Heats

Property to compare the energy storage capabilities of various substances

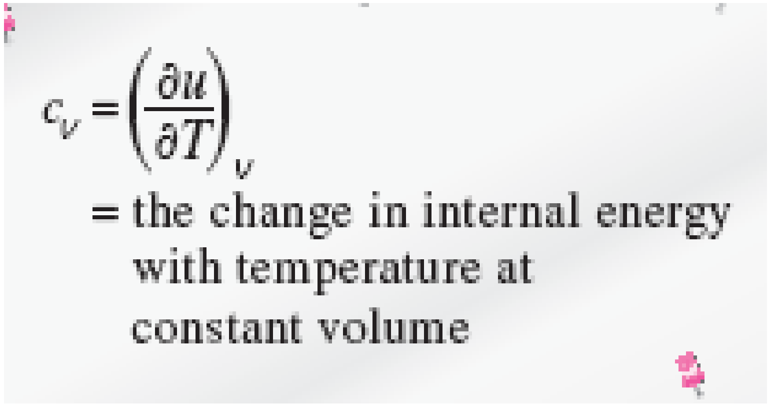
The energy required to raise a temperature of unit mass of a substance by one degree.



C_p : Specific heat at constant pressure
C_v : Specific heat at constant volume

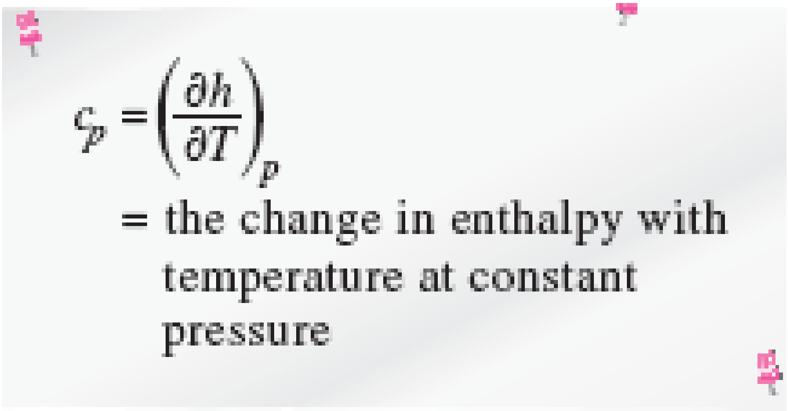
$$\boxed{C_p > C_v}$$

In thermodynamics, the specific heats are defined asc



$$c_v = \left(\frac{\partial u}{\partial T}\right)_v$$

= the change in internal energy with temperature at constant volume



$$c_p = \left(\frac{\partial h}{\partial T}\right)_p$$

= the change in enthalpy with temperature at constant pressure

Internal Energy, Enthalpy and Specific Heats of *Ideal Gas*

For an ideal gas the internal energy is a function of the temperature only. That is,

$$u = u(T)$$

Using the definition of enthalpy and the equation of state of an ideal gas, we have

$$h = u + Pv$$
$$Pv = RT$$
$$Then\ \ h = u + RT$$

Since R is constant and $u = u(T)$, it follows that the enthalpy of an ideal gas is also a function of temperature only:

$$h = h(T)$$

Then the differential changes in the internal energy and enthalpy of an ideal gas can be expressed as:

$$du = C_V\, dT$$

$$dh = C_P\, dT$$

The change in internal energy and enthalpy of ideal gases can be expressed as

$$\Delta u = u_2 - u_1 = \int_1^2 C_V(T)\,dT = C_{V,ave}(T_2 - T_1)$$

$$\Delta h = h_2 - h_1 = \int_1^2 C_P(T)\,dT = C_{P,ave}(T_2 - T_1)$$

where $C_{V,\,ave}$ and $C_{P,\,ave}$ are average or constant values of the specific heats over the temperature range

Relation between C_P and C_V for Ideal Gases

Using the definition of enthalpy ($h = u + Pv$) and writing the differential of enthalpy, the relationship between the specific heats for ideal gases is

$$
\begin{gathered}
h = u + Pv \\
dh = du + d(RT) \\
C_P\, dT = C_V\, dT + R dT \\
C_P = C_V + R
\end{gathered}
$$

where R is the particular gas constant.

When the specific heats are given on a molar basis, R in the above equation should be replaced by the universal gas constant R_u

$$\bar{c}_p = \bar{c}_v + R_u \qquad (\mathrm{kJ/kmol \cdot K})$$

The **specific heat ratio *k*** (fluids texts often use γ instead of k) is defined as

$$k = \frac{C_P}{C_V}$$

Example (6)

Air at 300 K and 200 kPa is heated at constant pressure to 600 K. Determine the change in internal energy of air per unit mass, using

(*a*) data from the air table (Table A–17), (*b*) the functional form of the specific heat (Table A–2*c*), and (*c*) the average specific heat value (Table A–2*b*).

Example (7)

An insulated rigid tank initially contains 0.7 kg of helium at 27°C and 350 kPa. A paddle wheel with a power rating of 0.015 kW is operated within the tank for 30 min. Determine

a) the final temperature

b) the final pressure of the helium gas.

Example (8)

A piston–cylinder device initially contains 0.5 m^3 of nitrogen gas at 400 kPa and 27°C. An electric heater within the device is turned on and is allowed to pass a current of 2 A for 5 min from a 120-V source. Nitrogen expands at constant pressure, and a heat loss of 2800 J occurs during the process. Determine the final temperature of nitrogen.

Example (9)

A piston–cylinder device initially contains air at 150 kPa and 27°C. At this state, the piston is resting on a pair of stops, as shown in Fig. 4–32, and the enclosed volume is 400 L. The mass of the piston is such that a 350-kPa pressure is required to move it. The air is now heated until its volume has doubled. Determine

(*a*) the final temperature, (*b*) the work done by the air, and (*c*) the total heat transferred to the air.

Internal Energy, Enthalpy, and Specific Heats of Solids and Liquids

The constant-volume and constant-pressure specific heats are identical for incompressible substances

$$c_p = c_v = c$$

Internal Energy Changes

The specific heats of incompressible substances depend on Temp. only.

$$du = C_V\,dT = C(T)\,dT$$

By integration

$$\Delta u = u_2 - u_1 = \int_1^2 c(T)\,dT$$

For small temperature intervals, a c value at the average temperature can be used and treated as a constant, yielding

$$\Delta u \cong c_{\text{avg}}(T_2 - T_1)$$

Enthalpy Changes

Using the definition of enthalpy $h = u + Pv$ and noting that $v=$ constant, the differential form of the enthalpy change of incompressible substances can be determined by differentiation to be

$$dh = du + v\,dP + P\,dv = du + v\,dP$$

Integrating

$$\Delta h = \Delta u + v\,\Delta P \cong c_{\text{avg}}\,\Delta T + v\,\Delta P$$

For **solids**, the term $v\,\Delta P$ is insignificant and thus

$$\Delta h_{solid} = \Delta u_{solid} \cong c_{avg}\,\Delta T$$

For **liquids**, two special cases are encountered:

Constant-pressure processes, as in heaters ($\Delta P = 0$)

$$\Delta h_{liquid} = \Delta u_{liquid} \cong c_{avg} \Delta T$$

Constant-temperature processes, as in pumps ($\Delta T = 0$)

$$\Delta h_{liquid} = \Delta u_{liquid} + v\Delta P \cong c_{avg} \Delta T + v\Delta P$$

$$\Delta h_{liquid} = v\Delta P$$

Example (10)

A 50-kg iron block at 80°C is dropped into an insulated tank that contains 0.5 m^3 of liquid water at 25°C. Determine the temperature when thermal equilibrium is reached.