Selection of Engineering Mat.s
Lec-6
Thermal& Electrical Charts

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The thermal conductivity–electrical resistivity chart

The material property governing the flow of heat through a material at steady state is the *thermal conductivity*, λ (units: W/m.K). (See Figure 4.10.) The valence electrons in metals are "free," moving like a gas within the lattice of the metal. Each electron carries a kinetic energy, $\frac{3}{2}kT$, where k is Boltzmann's constant. It is the transmission of this energy, via collisions, that conducts heat in metals. The thermal conductivity is described by

$$\lambda = \frac{1}{3} C_e \, \overline{c} \, \ell \tag{4.11}$$

where C_e is the electron specific heat per unit volume, $\bar{c}$ is the electron velocity (2×10^5 m/s), and ℓ is the electron mean-free path, typically 10^{-7} m in pure metals. In heavily alloyed solid solutions (stainless steels, nickel-based superalloys, and titanium alloys) the foreign atoms scatter electrons, reducing the mean free path to atomic dimensions ($\approx 10^{-10}$ m), much reducing λ.

These same electrons, when in a potential gradient, drift through the lattice, giving electrical conduction. The electrical conductivity, κ, is measured here

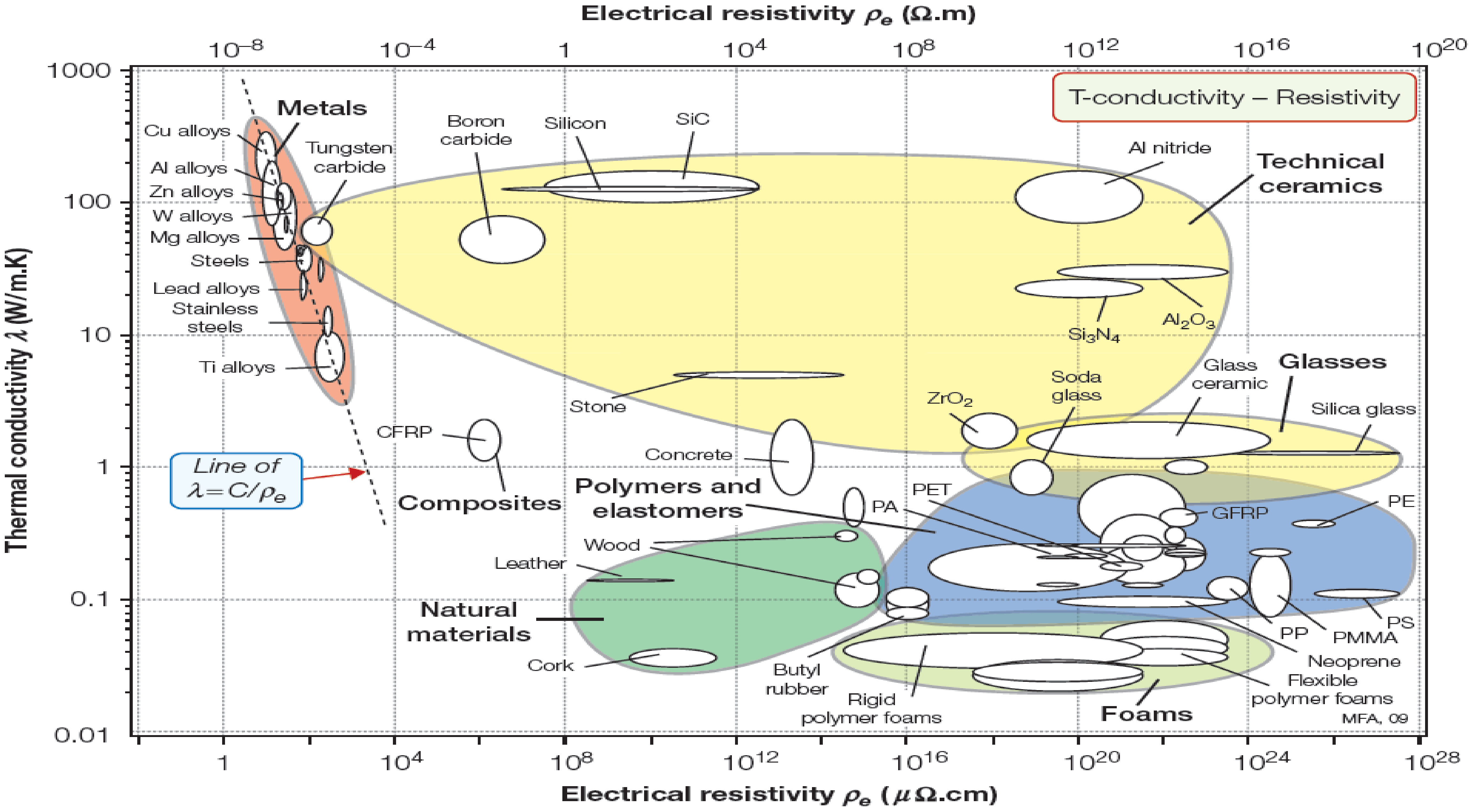


FIGURE 4.10

Thermal conductivity λ plotted against electrical resistivity ρ_e. For metals the two are related.

by its reciprocal, the *resistivity* ρ_e (SI units: Ω.m, units of convenience μΩ.cm). The range is enormous: a factor of 10^{28}, far larger than that of any other property. As with heat, the conduction of electricity in metals is proportional to the density of carriers, the electrons, and their mean-free path, leading to the Wiedemann-Franz relation

$$\lambda \propto \kappa = \frac{1}{\rho_e} \tag{4.12}$$

The quantities λ and ρ_e are the axes of Figure 4.10. Data for metals appear at the top left. The broken line shows that the Wiedemann-Franz relation is well obeyed.

But what about the rest of the chart? Electrons do not contribute to thermal conduction in ceramics and polymers. Heat is carried instead by phonons—lattice vibrations of short wavelength. They are scattered by each other and by impurities, lattice defects, and surfaces; it is these that determine the phonon mean-free path, ℓ. The conductivity is still given by equation (4.11), which we write as

$$\lambda = \frac{1}{3} \rho \, C_p \, \overline{c} \, \ell \tag{4.13}$$

but now $\bar{c}$ is the elastic wave speed (around 10^3 m/s—see Figure 4.3), ρ is the density, and C_p is the *specific heat per unit mass* (units: J/kg.K). If the crystal is particularly perfect and the temperature is well below the Debye temperature, as in diamond at room temperature, the phonon conductivity is high: It is for this reason that single crystal silicon carbide and aluminum nitride have thermal conductivities almost as high as copper.

The low conductivity of glass is caused by its irregular amorphous structure; the characteristic length of the molecular linkages (about 10^{-9} m) determines the mean free path. Polymers have low conductivities because the elastic wave speed $\bar{c}$ is low (see Figure 4.3), and the mean free path in the disordered structure is small. Highly porous materials like firebrick, cork, and foams show the lowest thermal conductivities, limited by the thermal conductivity of the gas in their cells.

Graphite and many intermetallic compounds such as C and B_4C, like metals, have free electrons, but the number of carriers is smaller and the resistivity is higher than in metals. Defects such as vacancies and impurity atoms in ionic solids create positive ions that require balancing electrons. These can jump from ion to ion, conducting charge, but slowly because the carrier density is low. Covalent solids and most polymers have no mobile electrons and are insulators ($\rho_e > 10^{12}\ \mu\Omega.\text{cm}$)—they lie on the right side of Figure 4.10.

Under a sufficiently high potential gradient, anything will conduct. The gradient tears electrons free from even the most possessive atoms, accelerating them into collision with nearby atoms, knocking out more electrons and creating a cascade. The critical gradient is the *breakdown potential*, V_b (units: MV/m), defined in Chapter 3.

Conducting heat but not electricity

Which materials are both good thermal conductors and good electrical insulators (an unusual combination)? Use Figure 4.10 to find out.

Answer

The chart identifies aluminum nitride, alumina, and silicon nitride as having these properties. They are the ones at the top right.

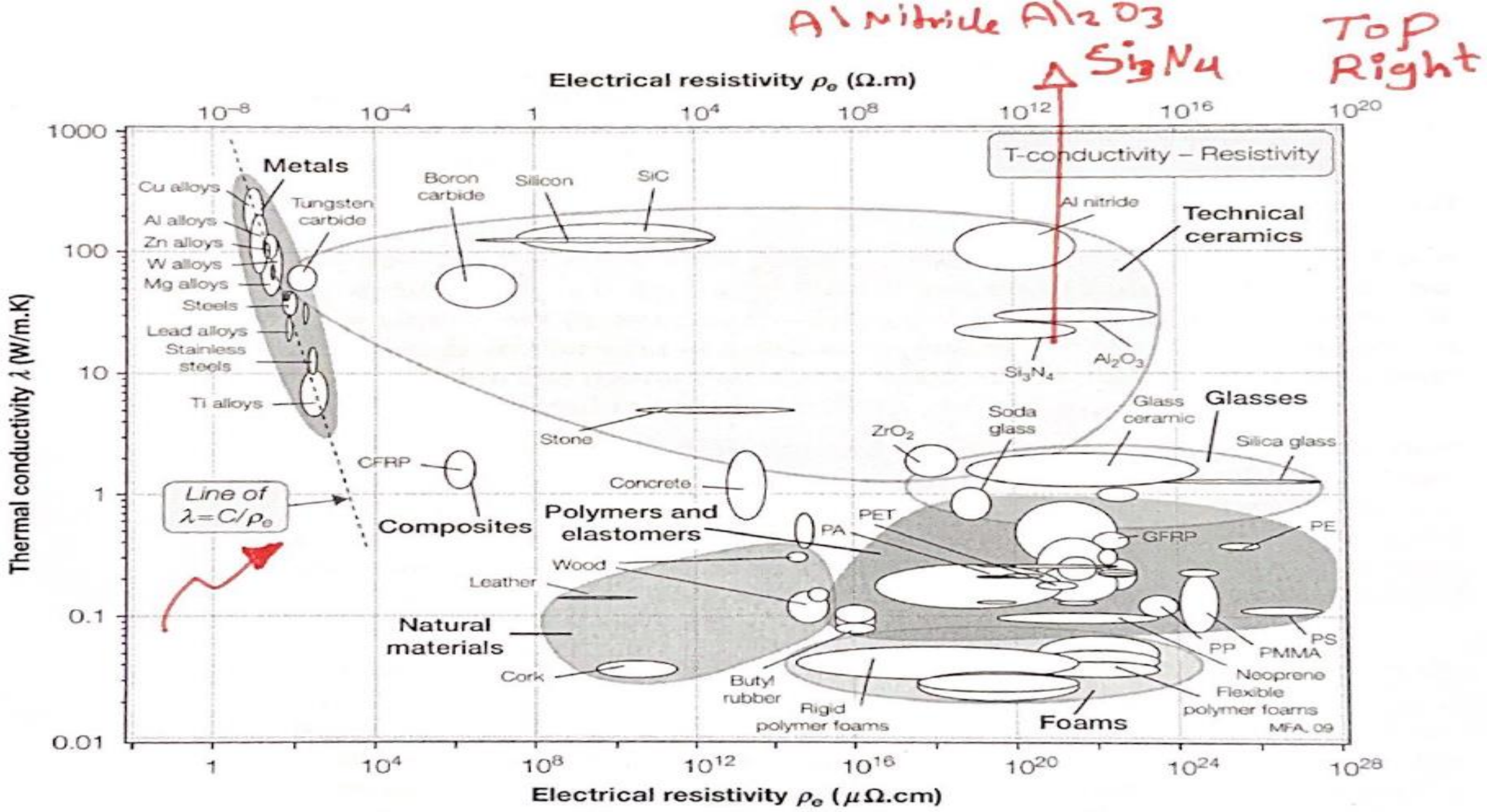


FIGURE 4.10

Thermal conductivity λ plotted against electrical resistivity ρ_e. For metals the two are related.

The thermal conductivity–thermal diffusivity chart

Thermal conductivity, as we have said, governs the flow of heat through a material at steady state. The property governing transient heat flow is the *thermal diffusivity*, a (units: m^2/s). The two are related by

$$a = \frac{\lambda}{\rho C_p} \qquad (4.14)$$

where ρ in kg/m^3 is the density. The quantity ρC_p is the *volumetric specific heat* (units: $J/m^3.K$). Figure 4.11 relates thermal conductivity, diffusivity, and volumetric specific heat, at room temperature.

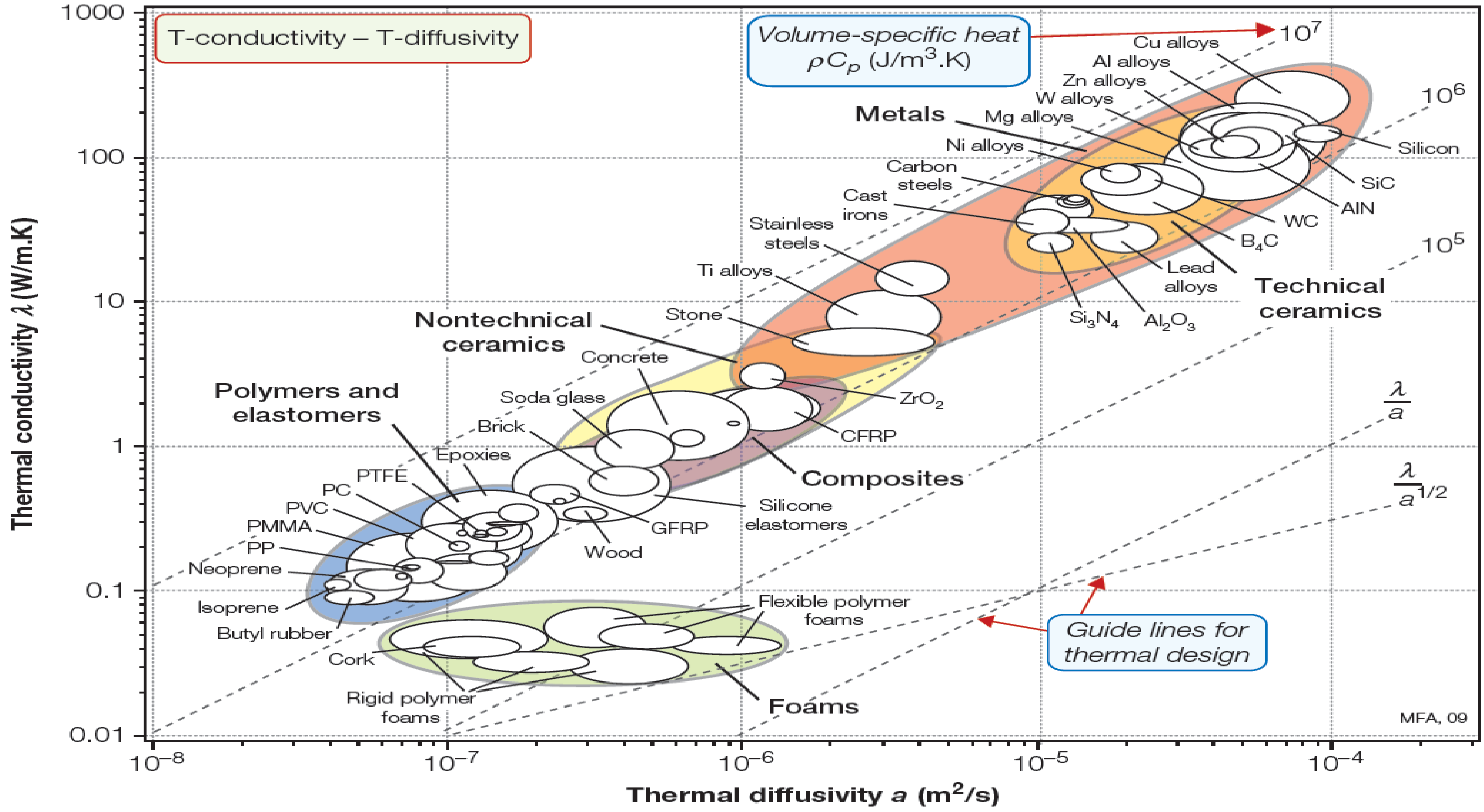


FIGURE 4.11

Thermal conductivity λ plotted against thermal diffusivity a. The contours show the volume-specific heat ρC_v. All three properties vary with temperature; the data here are for room temperature.

The data span almost five decades in λ and a. Solid materials are strung out along the line[3]

$$\rho C_p \approx 3\times 10^6 \ \text{J/m}^3\text{.K} \tag{4.15}$$

As a general rule, then,

$$\lambda = 3\times 10^6 a \tag{4.16}$$

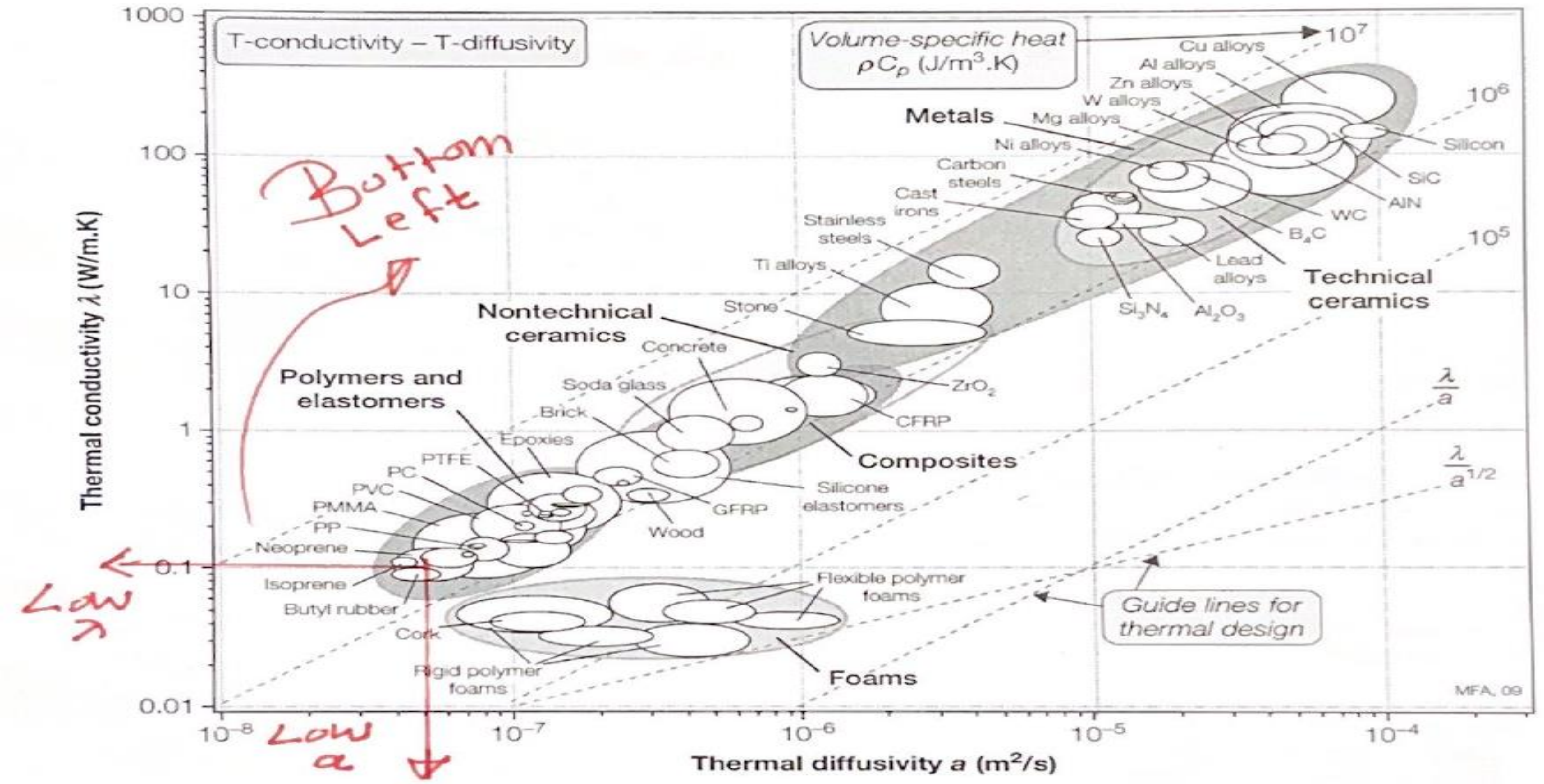
(λ in W/m.K and a in m^2/s). Some materials deviate from this rule because they have lower than average volumetric specific heat. The largest deviations are shown by porous solids: foams, low-density firebrick, woods, and the like. Their low density means that they contain fewer atoms per unit volume and, averaged over the volume of the structure, ρC_p is low. The result is that, although foams have low thermal *conductivities* and are widely used for insulation, their thermal *diffusivities* are not necessarily low: They may not transmit much heat, but they reach a steady state quickly. This is important in design, a point illustrated by the case study in Section 6.13.

Thermal buffers

A good way to protect equipment from sudden temperature change is to encase it in a material with a very low thermal diffusivity, because an external temperature change then takes a long time to reach the inside. Use Figure 4.11 to identify materials that could be good for this.

Answer

The chart identifies isoprene, neoprene, and butyl rubber as potential candidates.



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Thermal conductivity λ plotted against thermal diffusivity a. The contours show the volume-specific heat ρC_v. All three properties vary with temperature; the data here are for room temperature.

The thermal expansion–thermal conductivity chart

Almost all solids expand on heating (Figure 4.12). The bond between a pair of atoms behaves like a linear elastic spring when the relative displacement of the atoms is small, but when it is large the spring is nonlinear. Most bonds become stiffer when the atoms are pushed together and less stiff when they are pulled apart. Such bonds are anharmonic. The thermal vibrations of atoms, even at room temperature, involve large displacements; as

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[3] This can be understood by noting that a solid containing N atoms has $3N$ vibrational modes. Each (in the classical approximation) absorbs thermal energy kT at the absolute temperature T, and the vibrational specific heat is $C_p \approx C_v = 3Nk$ (J/K) where k is Boltzmann's constant (1.34×10^{-23} J/K). The volume per atom Ω for almost all solids lies within a factor of two of 1.4×10^{-29} m^3; thus the volume of N atoms is (NC_p) m^3. The volume-specific heat is then (as the chart shows):

$$\rho C_v \cong 3\,N\,k/N\,\Omega = \frac{3\,k}{\Omega} = 3 \times 10^6\ \mathrm{J/m^3K}$$

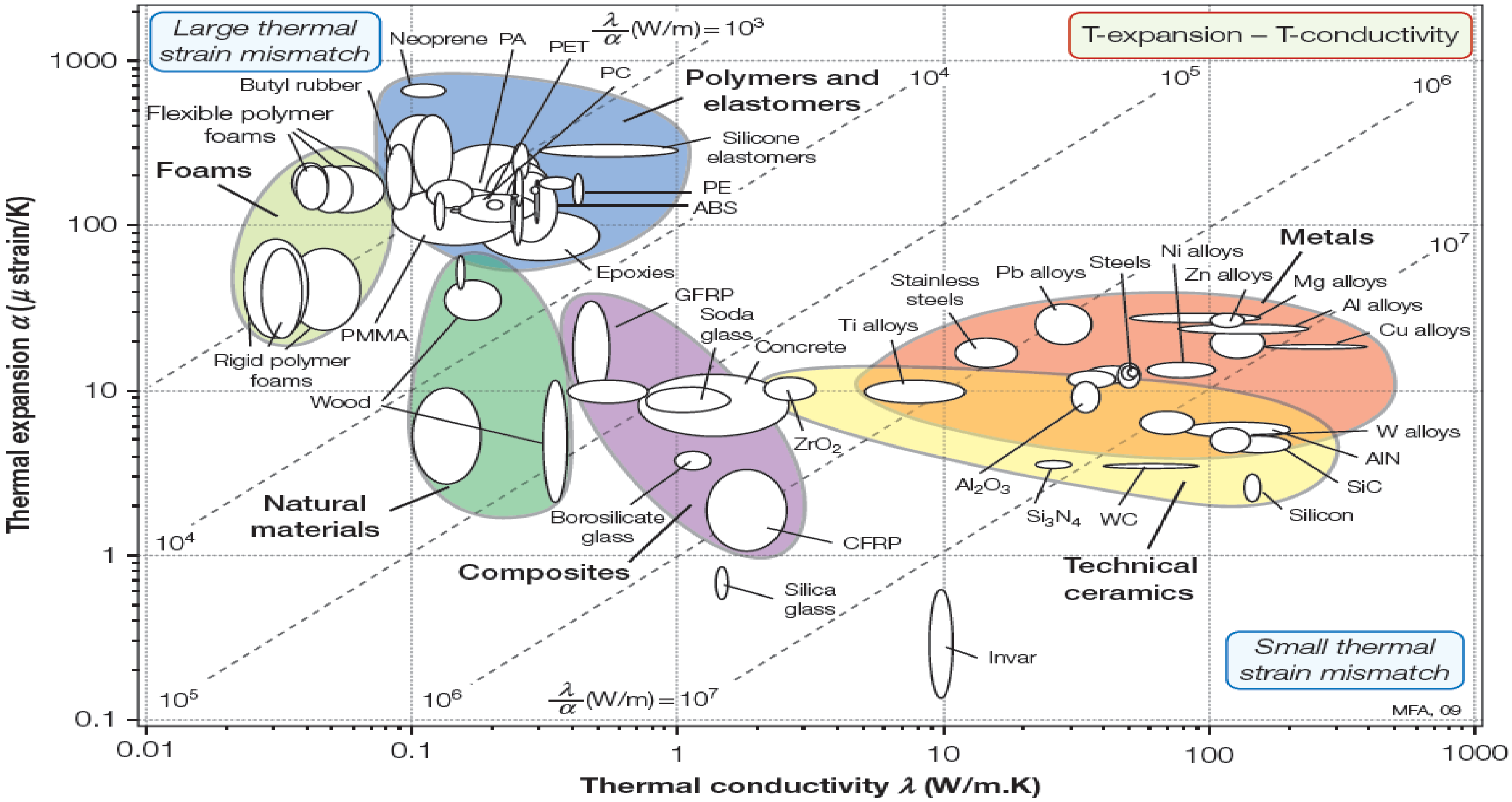


FIGURE 4.12

The linear expansion coefficient α plotted against the thermal conductivity λ. The contours show the thermal distortion parameter λ/α. An extra material, the nickel alloy Invar, has been added to the chart; it is noted for its exceptionally low expansion at and near room temperature, useful in designing precision equipment that must not distort if the temperature changes.

the temperature is raised, the anharmonicity of the bond pushes the atoms apart, increasing their mean spacing. The effect is measured by the linear *expansion coefficient*

$$\alpha = \frac{1}{\ell} \frac{d\ell}{dT} \tag{4.17}$$

where ℓ is a linear dimension of the body.

The expansion coefficient is plotted against the thermal conductivity in Figure 4.12. It shows that polymers have large values of α, roughly 10 times greater than those of metals and almost 100 times greater than those of ceramics. This is because the Van der Waals bonds of the polymer are very anharmonic. Diamond, silicon, and silica glass (SiO_2) have covalent bonds that have low anharmonicity (that is, they are almost linear elastic even at large strains), giving them low expansion coefficients. Composites, even though they have polymer matrices, can have low values of α because the reinforcing fibers, particularly carbon, expand very little.

The chart shows contours of λ/α, a quantity important in designing against thermal distortion. An extra material, Invar (a nickel alloy), has been added to the chart because of its uniquely low expansion coefficient at and near room temperature, a consequence of a trade-off between normal expansion and a contraction associated with a magnetic transformation. An application that uses the chart is developed in Chapter 6, Section 6.16.

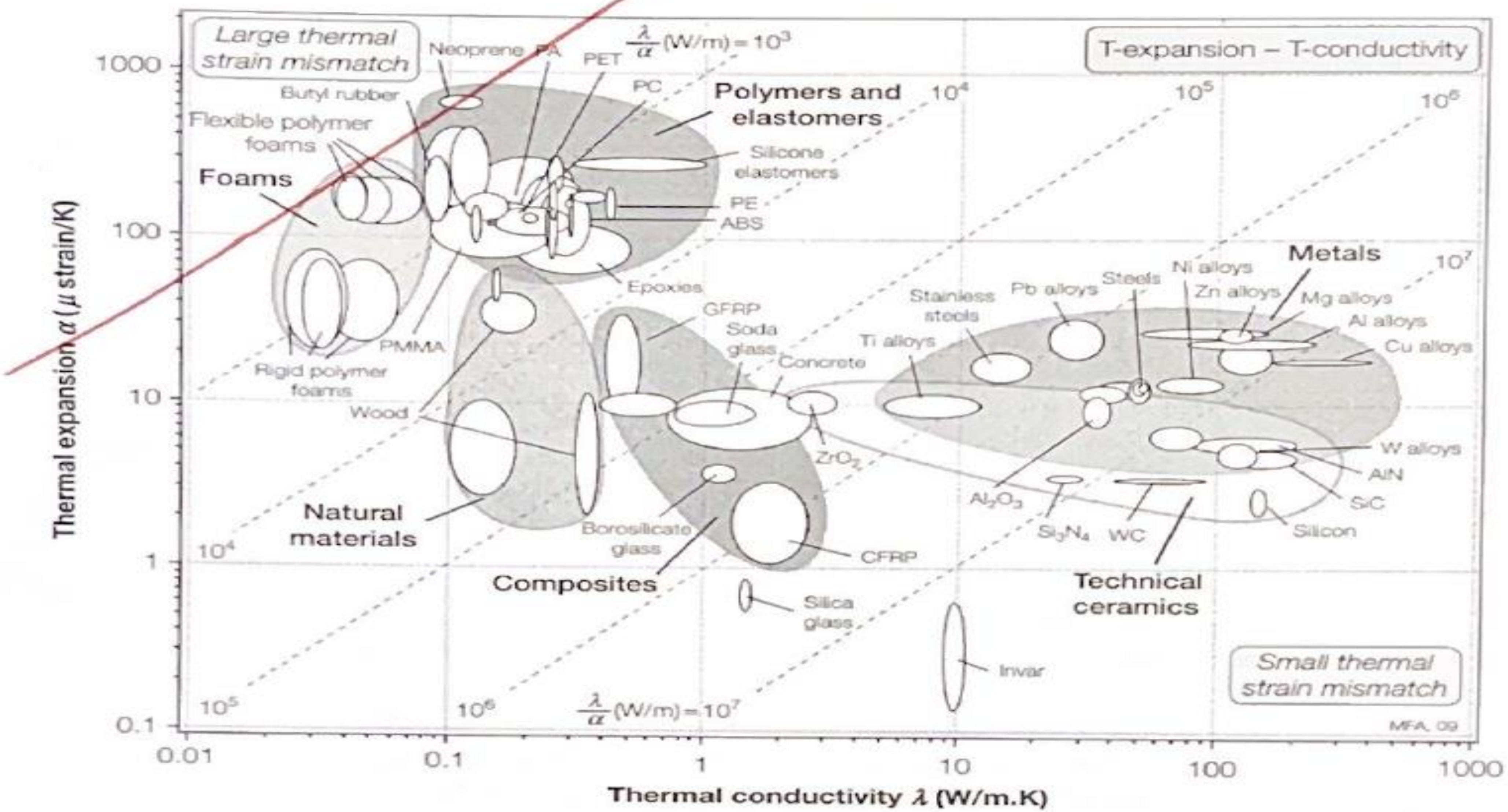
Thermal actuators

An actuator uses thermal expansion of its active element to generate the actuating force. Use Figure 4.12 to identify the material with the largest expansion coefficient.

Answer

Neoprene, at the upper left of the chart, has a larger value of expansion coefficient than any other on the chart.

Neoprene Upper Left



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