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Selection of Engineering Materials (CousreII) Lec-5

Entropy Based-Weight Aggregation Method in Materials Selection

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Introduction

Making decisions is a part of our daily routine. However, making the right decision has become more and more complicated as problems are growing in magnitude and longitude. More and more attributes of different alternatives must be considered or a group of decision-makers' judgment needs to be collected. Multiple-attribute group decision-making (MAGDM) might be the most common, but complex problem in the decision science field, which has been regarded as one of the most significant activities in industry, service, business, *etc*.

Preliminaries

In this section, an MAGDM problem is set up and its general solving steps are discussed. Then, we briefly review the entropy weighting technique and the principle of minimum cross-entropy.

2.1. Procedures of Solving MAGDM

A multiple-attribute group decision-making problem can be defined as a quadruple $< A, C, D, X >$, where:

$A = \{a_i| = 1, 2, \cdots, m\}$ is the alternative set for every decision-maker and is indexed by i and $m \geqslant 2$;

$C = \{c_j|j = 1, 2, \cdots, n\}$ is the attribute set for each alternative, and attributes are assumed to be additive and independent in this paper for simplicity;

$D = \{d_k|k = 1, 2, \cdots, l\}$ is the decision-maker set; and

$X = \{x_{ij}|i = 1, 2, \cdots, m; j = 1, 2, \cdots, n\}$ is the normalized value of the j-th attribute for the i-th alternative, *i.e.*,

$$
\boldsymbol{X} = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1n} \\ x_{21} & x_{22} & \cdots & x_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ x_{m1} & x_{m2} & \cdots & x_{mn} \end{bmatrix} \tag{1}
$$

The matrix $\boldsymbol{X}$ is the objective value of attributes. However, every decision-maker may have their own judgment on these values based on his or her preference. Hence, all decision-makers' judgment has to be integrated in order to solve an MAGDM problem. In this paper, we are going to introduce a utility function to express decision-makers' preference in accordance with the general approach in decision theory. Let $u_k(x)$ be the k-th decision-maker's utility function. Therefore, the problem confronted with the k-th decision-maker is:

$$
\boldsymbol{U}_k = u_k(\boldsymbol{X}) = \begin{bmatrix} u_k(x_{11}) & u_k(x_{12}) & \cdots & u_k(x_{1n}) \\ u_k(x_{21}) & u_k(x_{22}) & \cdots & u_k(x_{2n}) \\ \vdots & \vdots & \ddots & \vdots \\ u_k(x_{m1}) & u_k(x_{m2}) & \cdots & u_k(x_{mn}) \end{bmatrix}, \quad k = 1, 2, \cdots, l \tag{2}
$$

which can be viewed as a multiple attribute decision problem for the k-th decision-maker.

The entropy weighting technique is a widely-used method to determine the weight of an attribute based on the differences between them without any additional or subjective information. The differences are measured by information-theoretic entropy.

Generally speaking, multiple attribute decision-making has m alternatives, and each alternative has n attributes. Let r_{ij} be a non-negative value of the j-th attribute for the i-th alternative, such that a multiple attribute decision-making problem can be formalized into a matrix R as:

$$R = \begin{bmatrix} r_{11} & r_{12} & \cdots & r_{1n} \\ r_{21} & r_{22} & \cdots & r_{2n} \\ \cdots & \cdots & \ddots & \cdots \\ r_{m1} & r_{m2} & \cdots & r_{mn} \end{bmatrix} \tag{6}$$

In the entropy weighting technique, the entropy-based difference of the j-th attribute between alternatives is viewed as the foundation to determine the weight of attributes. When the difference of two alternatives about the j-th attribute is small, then this attribute does not provide sufficient information to rank or distinguish the two alternatives. Therefore, the less is the difference, the smaller is the weight. Mathematically, the weight of the j-th attribute in Equation (6) can be calculated out as:

$$\omega_j = \frac{1 - E_j}{\sum_{j=1}^{n}(1 - E_j)}, \quad j = 1, 2, \cdots, n \tag{7}$$

where E_j is an extended and normalized entropy defined as:

$$E_j = -\frac{1}{\ln m} \sum_{i=1}^{m} \frac{r_{ij}}{\sum_{i=1}^{m} r_{ij}} \ln \frac{r_{ij}}{\sum_{i=1}^{m} r_{ij}}, \quad j = 1, 2, \cdots, n \tag{8}$$

It is easy to find that $0 \leqslant \omega_j \leqslant 1$ and $\sum_{j=1}^{n} \omega_j = 1$ according to the properties of entropy.

Case-Study

Table 1 — Decision matrix.

Materials	P1	P2	P3
1	188	1.144	38
2	314	0.936	32
3	471	0.786	27
4	268	0.576	38
5	419	1.082	32
6	301	0.502	26
7	356	0.59	36
8	267	0.51	30
9	427	1.09	25
Maxi	**471**	**1.144**	**38**
Mini	**188**	**0.502**	**25**

Table 2 – Normalized decision matrix.

Materials	P1	P2	P3
1	0.3992	0.4388	0.6579
2	0.6667	0.5363	0.7813
3	1.0000	0.6387	0.9259
4	0.5690	0.8715	0.6579
5	0.8896	0.4640	0.7813
6	0.6391	1.0000	0.9615
7	0.7558	0.8508	0.6944
8	0.5669	0.9843	0.8333
9	0.9066	0.4606	1.0000
SUM	6.3928	6.2450	7.2935

Table 3 – Probability of the normalized decision matrix.

Materials	P1	P2	P3
1	0.0624	0.0703	0.0902
2	0.1043	0.0859	0.1071
3	0.1564	0.1023	0.1270
4	0.0890	0.1396	0.0902
5	0.1392	0.0743	0.1071
6	0.1000	0.1601	0.1318
7	0.1182	0.1362	0.0952
8	0.0887	0.1576	0.1143
9	0.1418	0.0737	0.1371

Table 4 – Entropy, divergence, and weights calculations.

	Materials	P1	P2	P3
		$Pr_{ij}\log_e$	(Pr_{ij})	
Materials	1	−0.1732	−0.1866	−0.2170
	2	−0.2357	−0.2108	−0.2393
	3	−0.2902	−0.2332	−0.2620
	4	−0.2153	−0.2748	−0.2170
	5	−0.2744	−0.1931	−0.2393
	6	−0.2302	−0.2933	−0.2671
	7	−0.2524	−0.2716	−0.2239
	8	−0.2148	−0.2912	−0.2479
	9	−0.2770	−0.1923	−0.2724
	Calculations			
I	$\sum_{i=1}^{n} Pr_{ij}\log_e(Pr_{ij})$	−2.1634	−2.1469	−2.1859
II	$Y = \frac{1}{\log_e(n)}$	0.4551	0.4551	0.4551
III	$En_j = -Y\sum_{i=1}^{n} Pr_{ij}\log_e(Pr_{ij})$	0.9846	0.9771	0.9948
IV	$Div_j = \lvert 1 - En_j \rvert$	0.0154	0.0229	0.0052
V	$\sum Div_j$	0.0424		
VI	Weight	0.3551	0.5273	0.1187
VII	Weight (%age)	35.51	52.73	11.87