MATHEMATICS I
FIRST SEMESTER

Lec. 01 Preliminaries

Outlines

- Basics
- Real Numbers and the Real Line
  - Real Numbers
  - Intervals
  - Solving Inequalities
  - Absolute Value
- Lines
  - *Cartesian Coordinates in the Plane*
  - Straight Lines (point-slope equation, slope-intercept equation)
  - Parallel and Perpendicular Lines
  - Distance in the Plane

Basics

Arithmetic Operations

$$a(b + c) = ab + ac$$

$$\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$$

$$\frac{a + c}{b} = \frac{a}{b} + \frac{c}{b}$$

$$\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc}$$

Factoring Special Polynomials

$$x^2 - y^2 = (x + y)(x - y)$$

$$x^3 + y^3 = (x + y)(x^2 - xy + y^2)$$

$$x^3 - y^3 = (x - y)(x^2 + xy + y^2)$$

Exponents and Radicals

$$x^m x^n = x^{m+n}$$

$$\frac{x^m}{x^n} = x^{m-n}$$

$$(x^m)^n = x^{mn}$$

$$x^{-n} = \frac{1}{x^n}$$

$$(xy)^n = x^n y^n$$

$$\left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}$$

$$x^{1/n} = \sqrt[n]{x}$$

$$x^{m/n} = \sqrt[n]{x^m} = \left(\sqrt[n]{x}\right)^m$$

$$\sqrt[n]{xy} = \sqrt[n]{x}\,\sqrt[n]{y}$$

$$\sqrt[n]{\frac{x}{y}} = \frac{\sqrt[n]{x}}{\sqrt[n]{y}}$$

Quadratic Formula

If $ax^2 + bx + c = 0$, then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Greek Alphabet

Greek name	Greek letter	
	Lower case	Capital
Alpha	α	A
Beta	β	B
Gamma	γ	Γ
Delta	δ	Δ
Epsilon	ϵ	E
Zeta	ζ	Z
Eta	η	H
Theta	θ	Θ
Iota	ι	I
Kappa	κ	K
Lambda	λ	Λ
Mu	μ	M

Greek name	Greek letter	
	Lower case	Capital
Nu	ν	N
Xi	ξ	Ξ
Omicron	o	O
Pi	π	Π
Rho	ρ	P
Sigma	σ	Σ
Tau	τ	T
Upsilon	υ	Υ
Phi	ϕ	Φ
Chi	χ	X
Psi	ψ	Ψ
Omega	ω	Ω

Real Numbers and the Real Line

● Real Numbers

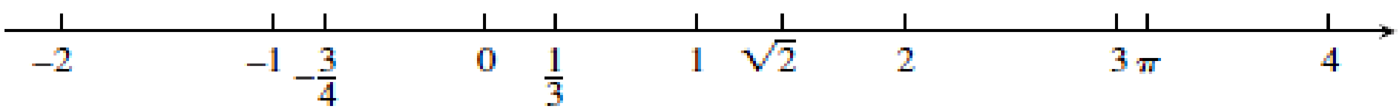
Much of calculus is based on properties of the real number system. **Real numbers** are numbers that can be expressed as decimals, such as

$$-\frac{3}{4} = -0.75000\ldots$$

$$\frac{1}{3} = 0.33333\ldots$$

$$\sqrt{2} = 1.4142\ldots$$

The real numbers can be represented geometrically as points on a number line called the **real line**.



The symbol $\mathbb{R}$ denotes either the real number system or, equivalently, the real line.

We distinguish three special subsets of real numbers.

1. The **natural numbers**, namely $1, 2, 3, 4, \ldots$
2. The **integers**, namely $0, \pm 1, \pm 2, \pm 3, \ldots$
3. The **rational numbers**, namely the numbers that can be expressed in the form of a fraction m/n, where m and n are integers and $n \neq 0$. Examples are

$$\frac{1}{3}, \quad -\frac{4}{9} = \frac{-4}{9} = \frac{4}{-9}, \quad \frac{200}{13}, \quad \text{and} \quad 57 = \frac{57}{1}.$$

● Intervals

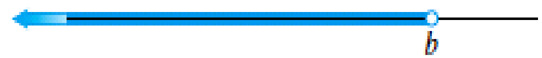
A subset of the real line is called an **interval** if it contains at least two numbers and contains all the real numbers lying between any two of its elements. For example, the set of all real numbers *x* such that $x > 6$ is an interval, as is the set of all *x* such that $-2 \leq x \leq 5$ The set of all nonzero real numbers is not an interval; since 0 is absent, the set fails to contain every real number between-1 and 1 (for example).

Geometrically, intervals correspond to rays and line segments on the real line, along with the real line itself. Intervals of numbers corresponding to line segments are **finite intervals**; intervals corresponding to rays and the real line are **infinite interval .**

● Solving Inequalities:

The process of finding the interval or intervals of numbers that satisfy an inequality in x is called solving the inequality.

TABLE 1.1 Types of intervals

	Notation	Set description	Type	Picture
Finite:	(a, b)	$\{x \mid a < x < b\}$	Open	
	$[a, b]$	$\{x \mid a \leq x \leq b\}$	Closed	
	$[a, b)$	$\{x \mid a \leq x < b\}$	Half-open	
	$(a, b]$	$\{x \mid a < x \leq b\}$	Half-open	
Infinite:	(a, ∞)	$\{x \mid x > a\}$	Open	
	$[a, \infty)$	$\{x \mid x \geq a\}$	Closed	
	$(-\infty, b)$	$\{x \mid x < b\}$	Open	
	$(-\infty, b]$	$\{x \mid x \leq b\}$	Closed	
	$(-\infty, \infty)$	$\mathbb{R}$ (set of all real numbers)	Both open	

EXAMPLE 1: Solve the following inequalities and show their solution sets on the real line.

a. $2x - 1 < x + 3$ b. $-\frac{x}{3} < 2x + 1$ c. $\frac{6}{x-1} \geq 5$

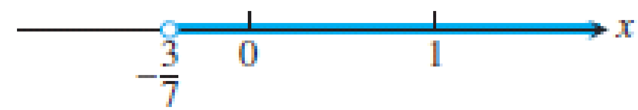
Solution:

a. $2x - 1 < x + 3 \rightarrow 2x < x + 4 \rightarrow x < 4$

The solution set is the open interval $(-\infty, 4)$

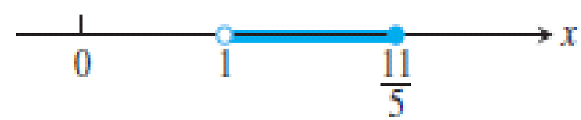
b. $-\frac{x}{3} < 2x + 1 \rightarrow -x < 6x + 3 \rightarrow 0 < 7x + 3$

$\rightarrow -3 < 7x \rightarrow -\frac{3}{7} < x$



The solution set is the open interval $(-\frac{3}{7}, \infty)$

C. $\frac{6}{x-1} \geq 5 \rightarrow 6 \geq 5x - 5 \rightarrow 11 \geq 5x \rightarrow \frac{11}{5} \geq x$



The inequality can hold only if $x > 1$,

Therefore, $(x - 1)$ is positive . solution set is the half-open interval $(1, 11/5)$

- ***Absolute Value:***

The **absolute value** of a number *x*, denoted by $|x|$ is defined by the formula.

$$|x| = \begin{cases} x, & x \geq o \\ -x & x < 0 \end{cases}$$

EXAMPLE 2: Finding Absolute Values

$|3| = 3$, $|0| = 0$, $|-5| = -(-5) = 5$, $|-|a|| = |a|$

$|x| = \sqrt{x^2}$, $|a| = \sqrt{a^2}$

Absolute Values and Intervals

If a is any positive number, then

- $|x| = a$ if and only if $x = \pm a$
- $|x| < a$ if and only if $-a < x < a$
- $|x| > a$ if and only if $x > a$ or $x < -a$
- $|x| \leq a$ if and only if $-a \leq x \leq a$
- $|x| \geq a$ if and only if $x \geq a$ or $x \leq -a$

Exercises 1.1

Solve the inequalities and show the solution sets on the real line.

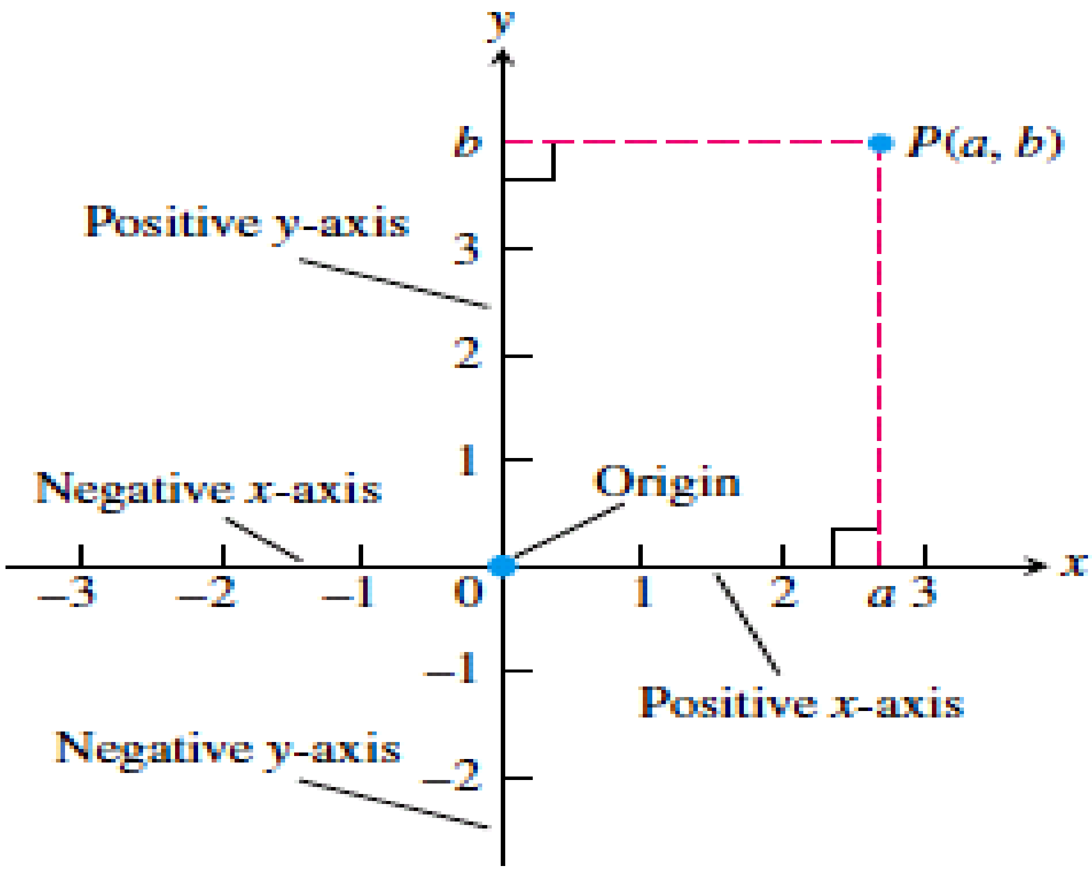
1. $3(2 - x) > 2(3 + x)$
2. $\frac{4}{5}(x - 2) < \frac{1}{3}(x - 6)$
3. $-\frac{x+5}{2} \leq \frac{12+3x}{4}$
4. $|\frac{x}{5} - 1| \leq 1$
5. $|3 - \frac{1}{x}| < \frac{1}{2}$
6. $|\frac{3x}{5} - 1| > \frac{2}{5}$
7. $|\frac{2x+7}{3}| \leq 5$
8. $(x - 1)^2 < 4$
9. $x^2 - x < 0$
10. $x^2 - x - 2 \geq 0$

Lines

”This section reviews coordinates, lines, and distance.

- ***Cartesian Coordinates in the Plane:***

draw two perpendicular coordinate lines that intersect at the 0-point of each line. These lines are called **coordinate axes** in the plane. On the horizontal x-axis, numbers are denoted by x increase to the right. On the vertical y-axis, numbers are denoted by y and increase upward *(Figure 1).* Thus “upward” and “to the right” are positive directions, whereas “downward” and “to the left” are considered as negative. The **origin** O, of the coordinate system is the point in the plane where x and y are both zero. If P is any point in the plane write $P(a, b)$. This coordinate system is called the **rectangular coordinate system** or **Cartesian coordinate system** this coordinate divide the plane into four regions called **quadrants**, as shown in *Figure 2.*



Figuer 1. Cartesian coordinates in the plane are based on two perpendicular axes intersecting at the origin

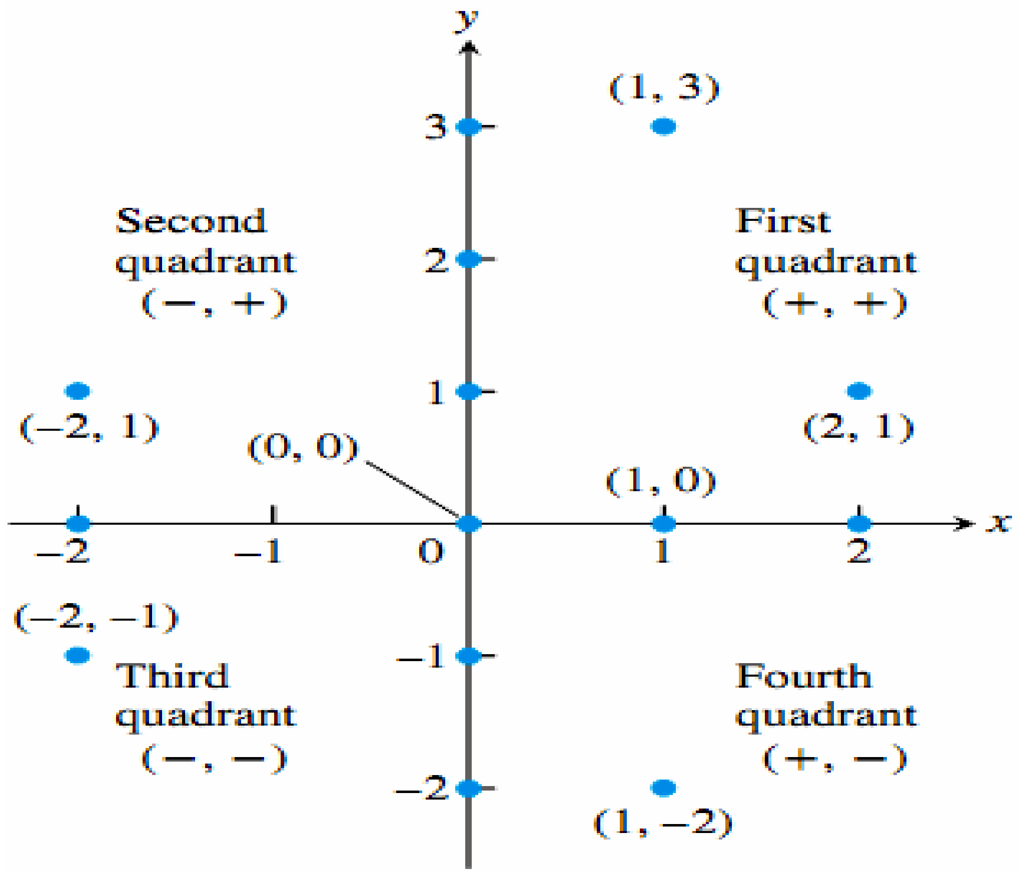


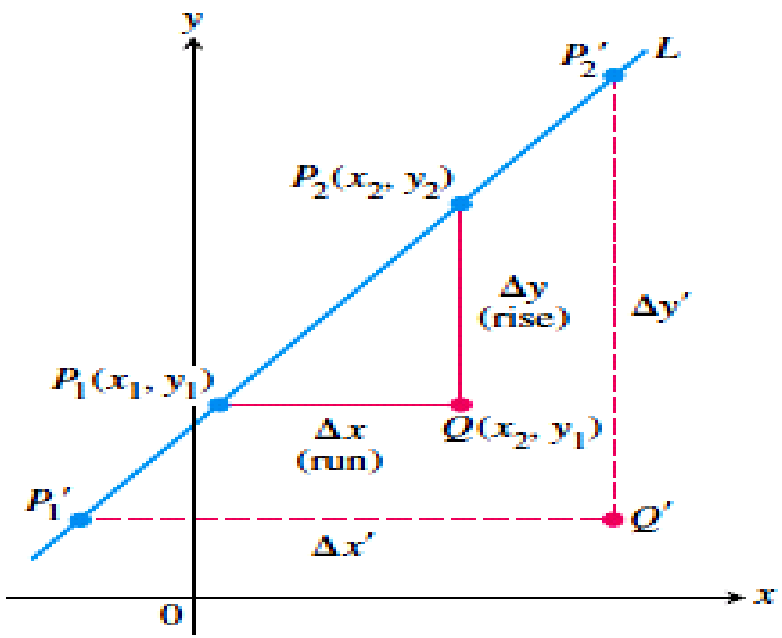
Figure 2. Points labeled in the xy-coordinate or Cartesian plane. The points on the axes all have coordinate pairs but are usually labeled with single real numbers, (so (1, 0) on the x-axis is labeled as 1). Notice the coordinate sign patterns of the quadrants.

• *Straight Lines (slop and equation):*

Slop: Given two points $P_1(x_1, y_1)$,and $P_2(x_2, y_2)$ in the plane, $\Delta x = x_2 - x_1$ and $\Delta y = y_2 - y_1$ the **run** and the **rise**, respectively, between P_1 and P_2 two such points straight line passing through them both. we call the line $P_1 P_2$. any nonvertical line in the plane has the property that the ratio .

the constant $$m = \frac{rise}{run} = \frac{\Delta y}{\Delta x} = \frac{y2-y1}{x2-x1}$$

is the **slope** of the nonvertical line $P_1 P_2$.



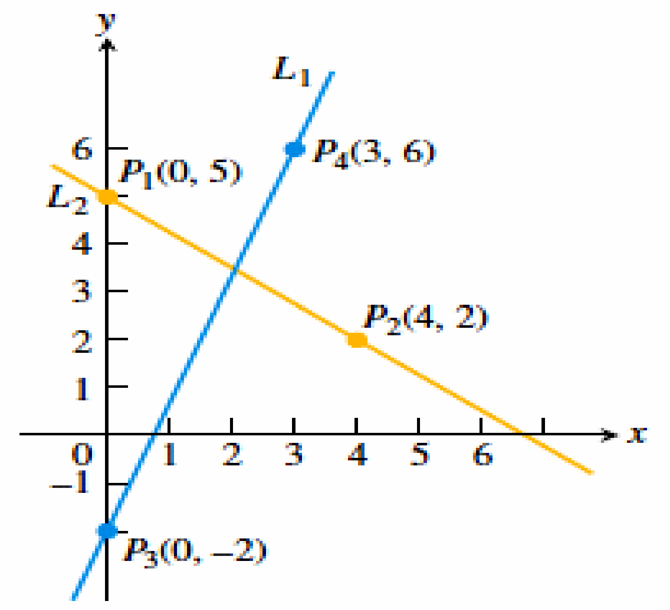
EXAMPLE 1: Find the slop of each line as shown.

solution: The slope of L1 is

$$m = \frac{\Delta y}{\Delta x} = \frac{6-(-2)}{3-0} = \frac{8}{3}$$

The slope of L2 is

$$m = \frac{\Delta y}{\Delta x} = \frac{2-5}{4-0} = \frac{-3}{4}$$



Straight lines have relatively simple equations. All points on the vertical line through the point a on the x-axis have x-coordinates equal to a. Thus, x=a is an equation for the vertical line. Similarly, y = b is an equation for the horizontal line meeting the y-axis at b.
(See Figure 3.)

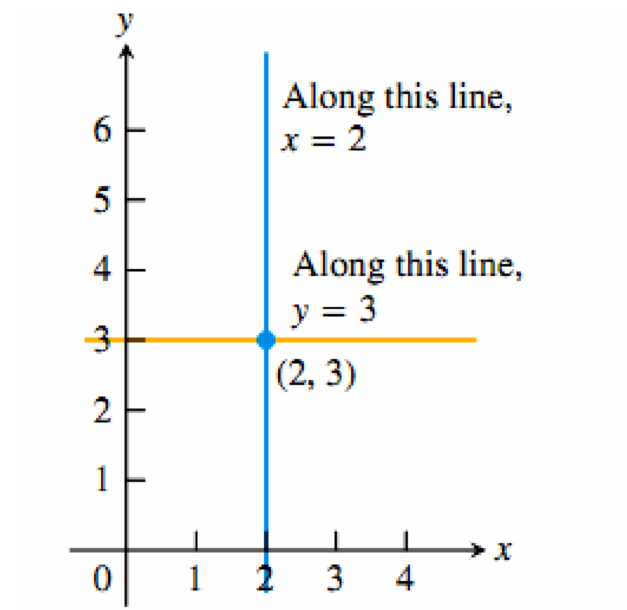


Figure 3 The standard equations for the vertical and horizontal lines through (2, 3) are and y = 3. x = 2

- ***point-slope equation:***

We can write an equation for a nonvertical straight line *L* if we know its slope *m* and the coordinates of one point$P_1(X_1,Y_1)$ on it. If $P(x, y)$ is *any* other point on *L*, then we can use the two points P_1 and *P* to compute the slope,

$m = \frac{y-y1}{x-x1}$ so that $y - y_1 = m(x - x_1)$ or $y = y_1 + m(x - x_1)$

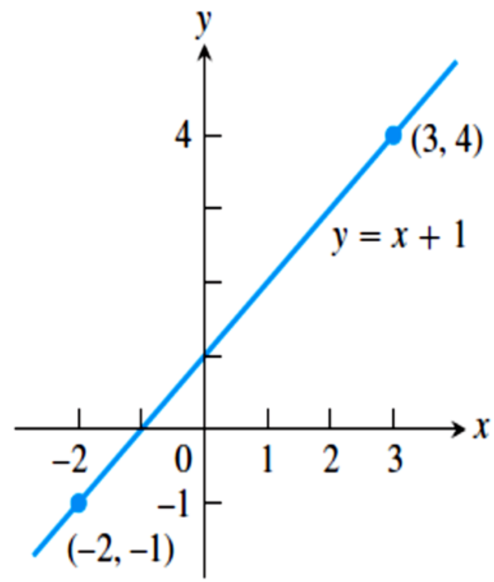
the equation $Y = Y_1 + m(X - X_1)$

is the **point-slope equation** of the line that passes through the point (X_1,Y_1) *and has slop m.*

EXAMPLE 2: Write an equation for the line through the point (2, 3) with slope -3/ 2 .

Solution: We substitute $x_1= 2$, $y_1=3$, and m=-3/ 2 into the point-slope equation and obtain $y = 3 - \frac{3}{2}(x - 2)$ *or* $y = -\frac{3}{2}x + 6$

EXAMPLE 3 A Line Through Two Points



Write an equation for the line through $(-2, -1)$ and $(3, 4)$.

Solution The line's slope is

$$m = \frac{-1 - 4}{-2 - 3} = \frac{-5}{-5} = 1.$$

We can use this slope with either of the two given points in the point-slope equation:

With $(x_1, y_1) = (-2, -1)$	**With $(x_1, y_1) = (3, 4)$**
$y = -1 + 1 \cdot (x - (-2))$	$y = 4 + 1 \cdot (x - 3)$
$y = -1 + x + 2$	$y = 4 + x - 3$
$y = x + 1$	$y = x + 1$

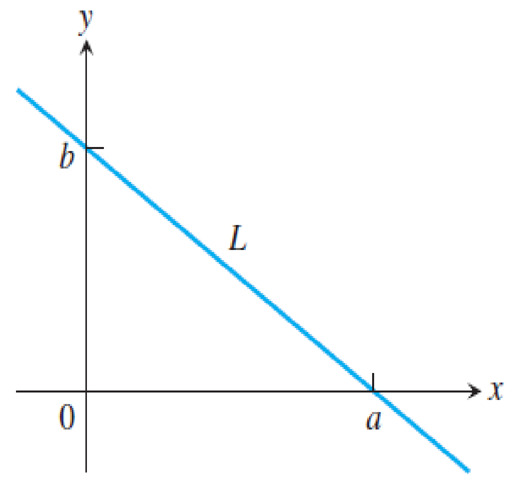
Same result

- ***Slope-intercept equation:***

The *y*-coordinate of the point where a nonvertical line intersects the *y*-axis is called the ***y* - intercept** of the line. Similarly, the ***x* - intercept** of the line .A line with slope *m* and *y*-intercept *b* passes through the point (0, *b*), so it has equation.

$$y = b + m(x - 0)$$

the equation $y = mx + b$ is called the slop-intercept equation of the line with slop m and y-intercept b.



Line *L* has *x*-intercept *a* and *y*-intercept *b*

Note: Lines with equations of the form y + mx have *y*-intercept 0 and so pass through the origin.

EXAMPLE 4:

Finding the Slope and *y*-Intercept Find the slope and *y*-intercept of the line $8x + 5y = 20$.

Solution: Solve the equation for *y* to put it in slope-intercept form:

$$8x + 5y = 20 \quad \rightarrow 5y = -8x + 20 \rightarrow y = -\frac{8}{5}x + 4$$

The slope is $m = -\frac{8}{5}$. The *y*-intercept is $b = 4$.

- **Parallel and Perpendicular Lines:**

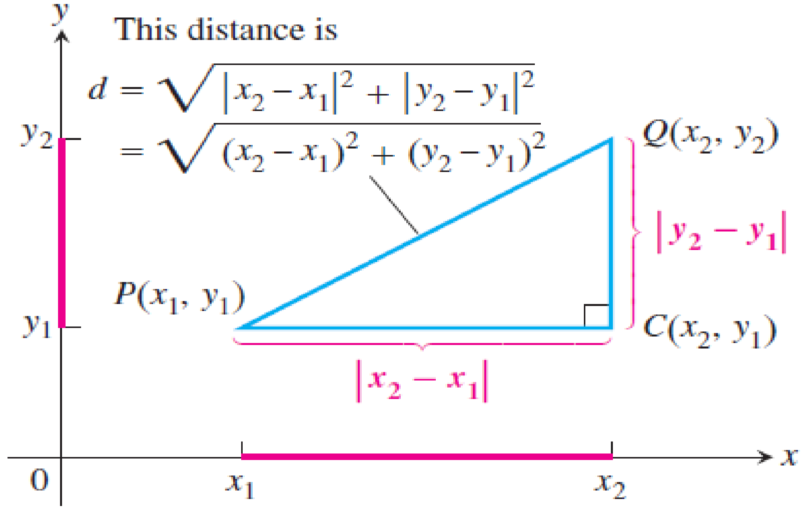
Lines that are parallel, so they have the same slope. $m_1 = m_2$

If two nonvertical lines L_1 and L_2 are perpendicular, their slopes m_1 and m_2 satisfy $m_1 m_2 = -1$, so each slope is the *negative reciprocal* of the other:

$$m_1 = -\frac{1}{m_2} \quad , \quad m_2 = -\frac{1}{m_1}$$

Distance and Circles in the Plane

The distance between points in the plane is calculated with a formula that comes from the Pythagorean theorem



calculate the distance between $P(x_1, y_1)$ and $Q(x_2, y_2)$, apply the Pythagorean theorem to triangle PCQ.

The distance between $P(x_1, y_1)$ and $Q(x_2, y_2)$ is

$$d = \sqrt{(\Delta x)^2 + (\Delta y)^2} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

EXAMPLE 5 Calculating Distance

(a) The distance between $P(-1, 2)$ and $Q(3, 4)$ is

$$\sqrt{(3 - (-1))^2 + (4 - 2)^2} = \sqrt{(4)^2 + (2)^2} = \sqrt{20} = \sqrt{4 \cdot 5} = 2\sqrt{5}.$$

(b) The distance from the origin to $P(x, y)$ is

$$\sqrt{(x - 0)^2 + (y - 0)^2} = \sqrt{x^2 + y^2}.$$

Exercises1.2:

Write an equation for each line described.

1. Passes through (-8,0) and (-1,3) .
2. Passes through (2,-3)with slop $1/2$.
3. Has slope $-5/4$ and *y*-intercept 6 .
4. Passes through (-12, -9) and has slope 0 .
5. Passes through ($1/3$, 4), and has no slope .
6. Has *y*-intercept -6 and *x*-intercept 2 .
7. Passes through($-\sqrt{2}$, 2)parallel to the line $\sqrt{2}\,x + 5y = \sqrt{3}$.
8. Passes through(4,10) and is perpendicular to the line 6x - 3y=5 .