الجامعة التكنولوجية
قسم هندسة المواد
Department of Materials Engineering

# Materials
# Lec-3
# Materials Indices for Elastic Design

**By**

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# Material indices for elastic design

material index $M_t$ (subscript '$t$' for tie) as:

$$M_t = \frac{E}{\rho} \quad \textit{specific stiffness}$$

the material index $M_p$ for the panel:

$$M_p = \frac{E^{1/3}}{\rho}$$

the mater index $M_b$ for the beam:

$$M_b = E^{1/2} / \rho = \text{constant}, C$$

the index for minimum material cost will be $M = E^{1/2} / (\rho C_m)$

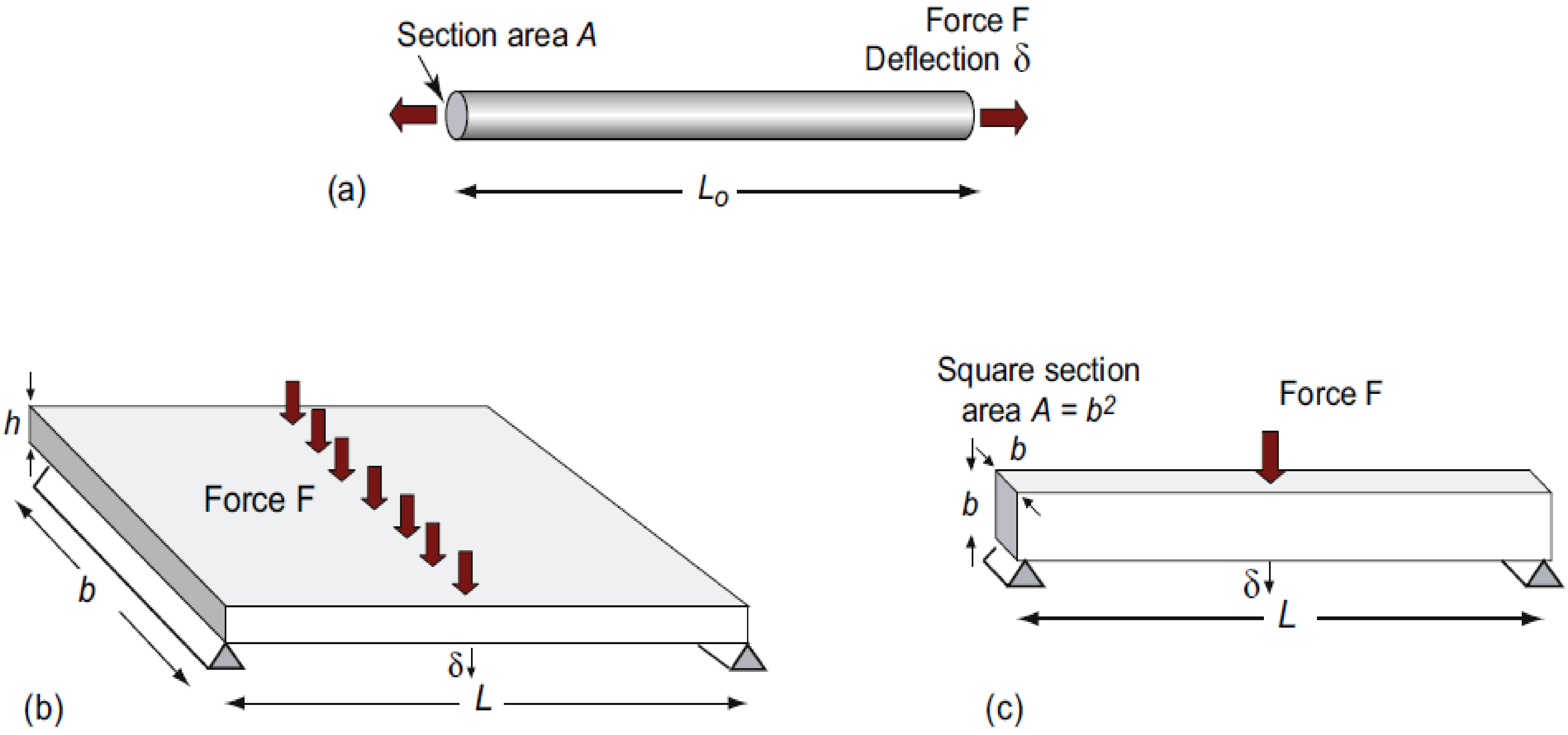


Figure 5.7 (a) A tensile tie; (b) a panel loaded in bending; (c) a beam of square section loaded in bending. In all cases, the stiffness $S = F/\delta$, where $F$ is the load, and $\delta$ is the extension or deflection.

***Ranking: indices on charts*** Consider first the material index for a light, stiff component loaded in tension (equation (5.16)):

$$M_t = E/\rho = \text{constant}, C$$

that is, for a given stiffness, equal mass corresponds to a constant value $C$ of the index (here the *specific stiffness*, $E/\rho$). Taking logs,

$$\log(E) = \log(\rho) + \log(C) \tag{5.31}$$

On the $\log(E)$–$\log(\rho)$ property chart, this is the equation of a straight line of slope 1. Similarly, for the light, stiff panel loaded in bending, the material index was (equation (5.22)):

$$M_p = E^{1/3} \big/ \rho = \text{constant}, C$$

which becomes, on taking logs:

$$\log(E) = 3\log(\rho) + \log(C) \tag{5.32}$$

This is another straight line, this time with a slope of 3. And by inspection, the third index (equation (5.27)) for a self-similar beam loaded in bending, $E^{1/2}/\rho$, plots as a line of slope 2.

Figure 5.10 shows a schematic of the $E-\rho$ chart with all three of the indices $E/\rho$, $E^{1/3}/\rho$ and $E^{1/2}/\rho$ plotted onto it – we refer to these lines as *selection guidelines*. It is now easy to read off a shortlist of materials that maximise performance for each loading geometry. For example, all the materials that lie on a line of constant $M = E^{1/3}/\rho$ perform equally well as a light, stiff panel; those above the line perform better, those below less well. Each selection guideline gives the slope of a family of parallel lines associated with each index. Figure 5.11 shows a grid of lines corresponding to values of $M_p$. Materials lying along the line with $M_p = 4.6$ all have the same mass, and this mass is 1/10 that of the materials lying along the line with $M_p = 0.46$. Each time we move the line to a higher value of $M_p$, the shortlist is reduced until we are left with the lightest options. The case studies in the next section give practical examples.

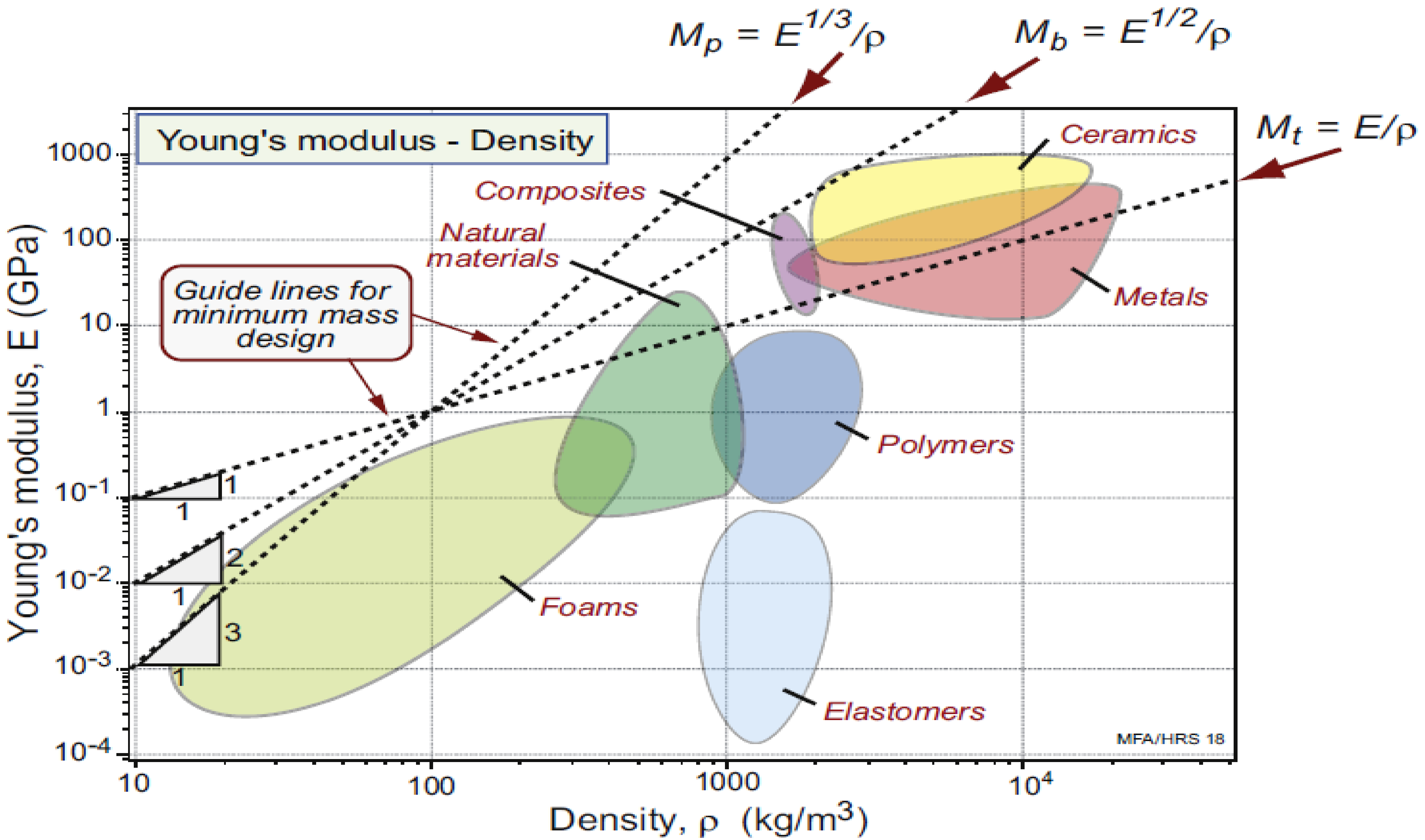
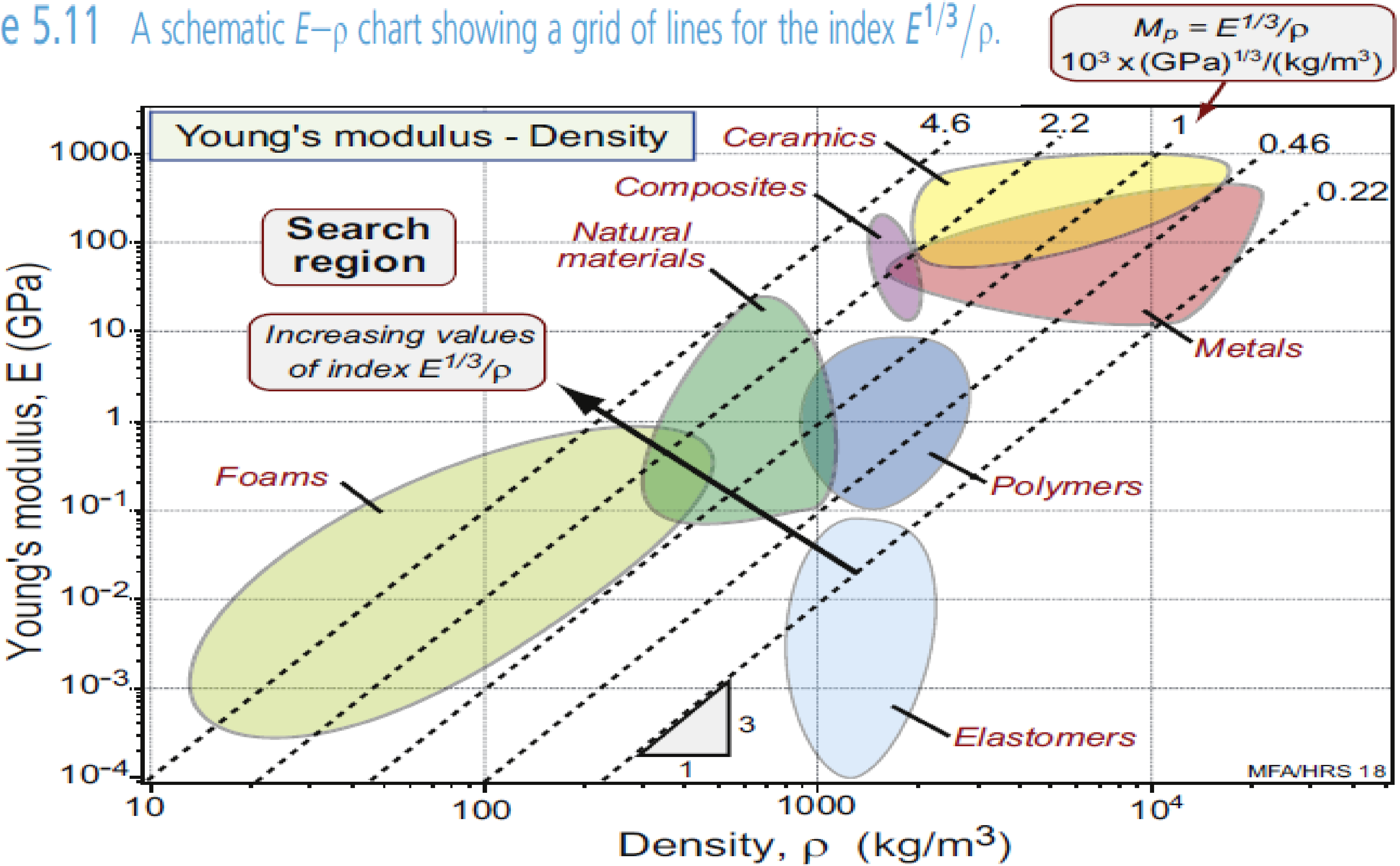

$M_p = E^{1/3}/\rho$
$M_b = E^{1/2}/\rho$
$M_t = E/\rho$
Young's modulus - Density
Ceramics
Composites
Natural materials
Metals
Guide lines for minimum mass design
Polymers
Foams
Elastomers
1
1
2
1
3
1
MFA/HRS 18
Young's modulus, E (GPa)
1000
100
10
1
$10^{-1}$
$10^{-2}$
$10^{-3}$
$10^{-4}$
10
100
1000
$10^4$
Density, $\rho$ (kg/m$^3$)

Figure 5.11 A schematic $E$–ρ chart showing a grid of lines for the index $E^{1/3}/\rho$.



## Plotting limits and indices on charts

*Screening: attribute limits on charts* Any design imposes certain non-negotiable demands (constraints) on the material of which it is made. Simple property limits can be plotted as horizontal or vertical lines on material property charts, as illustrated in Figure 5.9, which shows a schematic of the $E$–$\rho$ chart introduced in earlier chapters. Suppose that a design imposes limits of $E > 10$ GPa and $\rho < 2000\ \mathrm{kg/m^3}$, as shown in the figure. All materials in the window defined by the limits, labelled 'Search region', meet both constraints. Later chapters show charts for many other properties, allowing limits to be imposed, to narrow the shortlist. But simple limits are not appropriate for the properties contained within a material index, since a simple search window misses the trade-off between the key properties. So in the context of light, stiff design, we wish to rank materials against the objective of minimum mass for a given stiffness constraint. For this, we use the material indices derived in Section 5.3, plotted on suitable property charts.

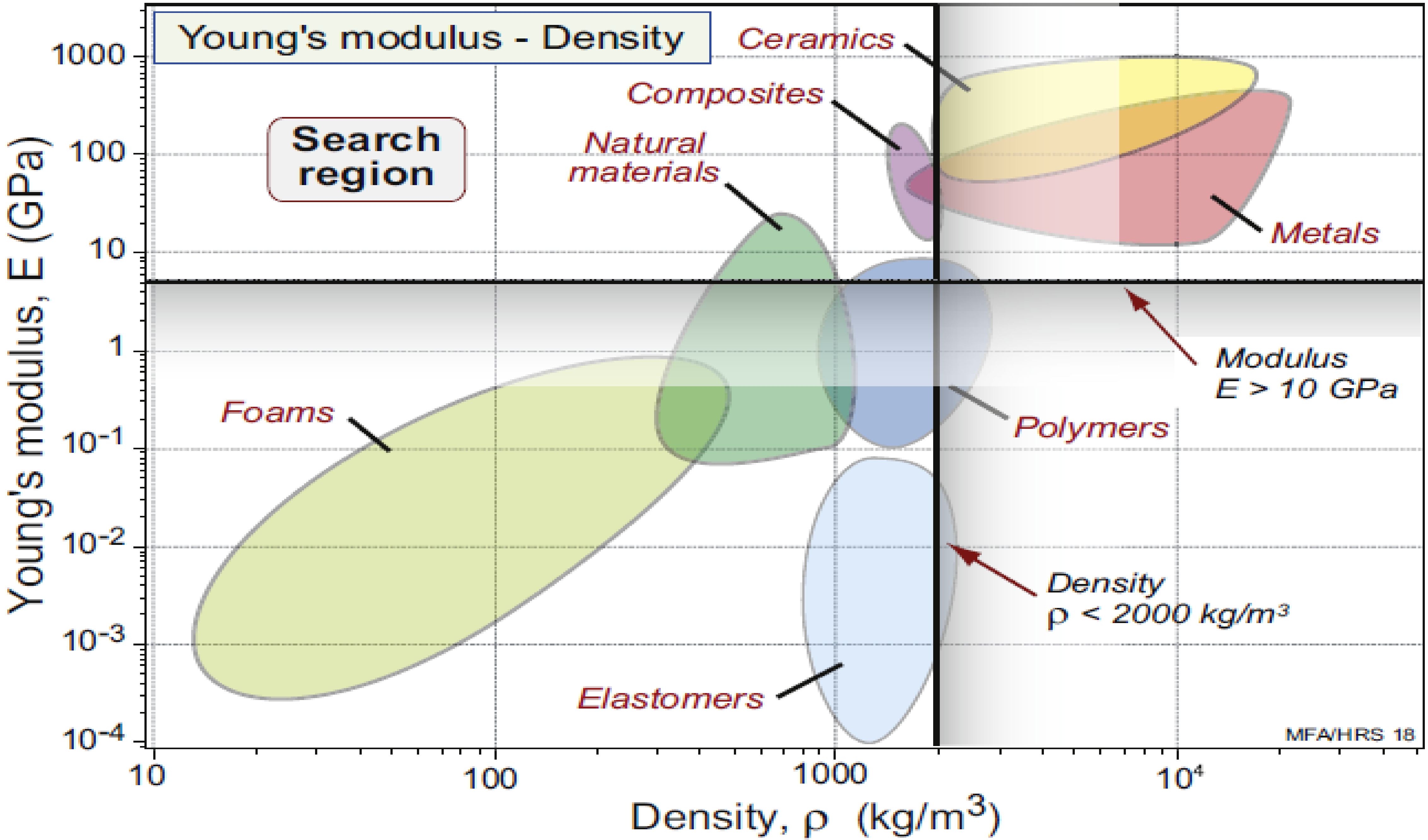

Young's modulus - Density
Search region
Ceramics
Composites
Natural materials
Metals
Modulus E > 10 GPa
Foams
Polymers
Density ρ < 2000 kg/m³
Elastomers
MFA/HRS 18
Young's modulus, E (GPa)
1000
100
10
1
10⁻¹
10⁻²
10⁻³
10⁻⁴
10
100
1000
10⁴
Density, ρ (kg/m³)

Figure 5.13 shows the appropriate chart: that in which Young's modulus, $E$, is plotted against density, $\rho$. The selection line for the index $M$ has a slope of 2, as explained in Section 5.3; it is positioned so that a small group of materials is left above it. They are the materials with the largest values of $M$, and it is these that are the best choice, provided they satisfy the other constraints. Three classes of materials lie above the line: woods, carbon fibre-reinforced polymers (CFRPs) and a number of ceramics. Ceramics are brittle and expensive, ruling them out. The recommendation is clear. Make the lever out of wood or – better – out of CFRP.

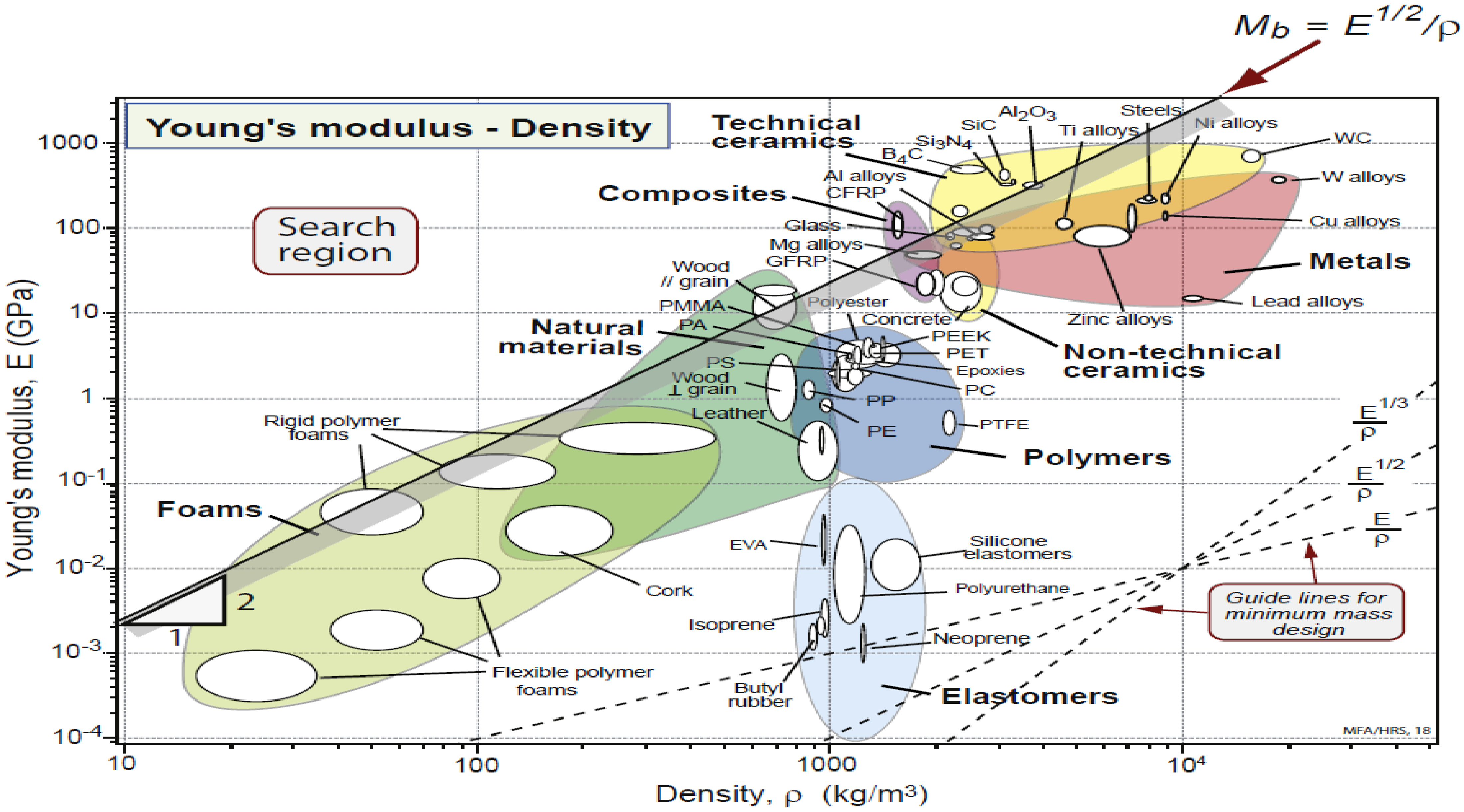


**Figure 5.13** Selection of materials for the corkscrew lever. The objective is to make it as light as possible while meeting a stiffness constraint.

# corkscrew lever

A corkscrew lever must be adequately stiff and, for travelling, be as light as possible.