الجامعة التكنولوجية
قسم هندسة المواد
Department of Materials Engineering

Selection of Engineering Materials (CousreII) Lec-2

THE ANALYTIC HIERARCHY PROCESS (AHP)

By

Prof.Dr.(Eng.) Abbas Khammas Hussein

2023-2024

INTRODUCTION

The Analytic Hierarchy Process (AHP) is due to Saaty (1980) and is often referred to, eponymously, as the Saaty method. It is popular and widely used, especially in military analysis, though it is not, by any stretch of the imagination, restricted to military problems. In fact, in his book, (which is not for the mathematically faint of heart!) Saaty describes case applications ranging from the choice of a school for his son, through to the planning of transportation systems for the Sudan. There is much more to the AHP than we have space for but we will cover the most easily used aspects of it.

The AHP deals with problems of the following type.

A firm wishes to buy one new piece of equipment of a certain type and has four aspects in mind which will govern its purchasing choice: expense, E; operability, O; reliability, R; and adaptability for other uses, or flexibility, F. Competing manufacturers of that equipment have offered three options, X, Y and Z. The firm's engineers have looked at these options and decided that X is cheap and easy to operate but is not very reliable and could not easily be adapted to other uses. Y is somewhat more expensive, is reasonably easy to operate, is very reliable but not very adaptable. Finally, Z is very expensive, not easy to operate, is a little less reliable than Y but is claimed by the manufacturer to have a wide range of alternative uses. (This is obviously a hypothetical example and, to understand Saaty properly, you should think of another case from your own experience.)

This is clearly an important and common class of problem and the AHP has numerous applications but also some limitations which will be discussed at the end of this section.

Before giving some worked examples of the AHP, we need first to explain the underlying ideas. You do not need to understand matrix algebra to follow the line of argument but you will need that mathematical ability actually to apply the AHP. Take heart, this is the only part of the book which uses any mathematics.

THE BASIC PRINCIPLES OF THE AHP

The mathematics of the AHP and the calculation techniques are briefly explained in Annex A but its essence is to construct a matrix expressing the relative values of a set of attributes. For example, what is the relative importance to the management of this firm of the cost of equipment as opposed to its ease of operation? They are asked to choose whether cost is very much more important, rather more important, as important, and so on down to very much less important, than operability. Each of these judgements is assigned a number on a scale. One common scale (adapted from Saaty) is:

The first step in the AHP procedure is to make pair wise comparisons between each criterion.

The example scale for comparison (Saaty & Vargas, 1991).

Scale	Degree of preference
1	Equal importance
3	Moderate importance of one factor over another
5	Strong or essential importance
7	Very strong importance
9	Extreme importance
2,4,6,8	Values for inverse comparison

Results of the comparison (for each factors pair) were described in term of integer values from 1 (equal value) to 9 (extreme different) where higher number means the chosen factor is considered more important in greater degree than other factor being compared with.

A basic, but very reasonable, assumption is that if attribute A is absolutely more important than attribute B and is rated at 9, then B must be absolutely less important than A and is valued at 1/9.

These pairwise comparisons are carried out for all factors to be considered, usually not more than 7, and the matrix is completed. The matrix is of a very particular form which neatly supports the calculations which then ensue (Saaty was a *very* distinguished mathematician).

The next step is the calculation of a list of the relative weights, importance, or value, of the factors, such as cost and operability, which are relevant to the problem in question (technically, this list is called an eigenvector). If, perhaps, cost is very much more important than operability, then, on a simple interpretation, the cheap equipment is called for though, as we shall see, matters are not so straightforward. The final stage is to calculate a Consistency Ratio (CR) to measure how consistent the judgements have been relative to large samples of purely random judgements. If the CR is much in excess of 0.1 the judgements are untrustworthy because they are too close for comfort to randomness and the exercise is valueless or must be repeated. It is easy to make a minimum number of judgements after which the rest can be calculated to enforce a perhaps unrealistically perfect consistency.

The AHP is sometimes sadly misused and the analysis stops with the calculation of the eigenvector from the pairwise comparisons of relative importance (sometimes without even computing the CR!) but the AHP's true subtlety lies in the fact that it is, as its name says, a *Hierarchy* process. The first eigenvector has given the relative importance attached to requirements, such as cost and reliability, but different machines contribute to differing extents to the satisfaction of those requirements. Thus, subsequent matrices can be developed to show how X, Y and Z respectively satisfy the needs of the firm. (The matrices from this lower level in the hierarchy will each have their own eigenvectors and CRs.) The final step is to use standard matrix calculations to produce an overall vector giving the answer we seek, namely the relative merits of X, Y and Z vis-à-vis the firm's requirements.

Now, calculate the consistency ratio and check its value.

- The purpose for doing this is to make sure that the original preference ratings were consistent.

There are 3 steps to arrive at the consistency ratio:

1.Calculate the consistency measure.

2.Calculate the consistency index (CI).

$$CI = \frac{\lambda max - n}{n - 1}.$$

3.Calculate the consistency ratio (CI/RI where RI is a random index).

$$CR = CI / RI$$

Random Consistency Index (RI)

n	1	2	3	4	5	6	7	8	9	10
RI	0	0	0.58	0.9	1.12	1.24	1.32	1.41	1.45	1.49

Then, he proposed what is called Consistency Ratio, which is a comparison between Consistency Index and Random Consistency Index, or in formula

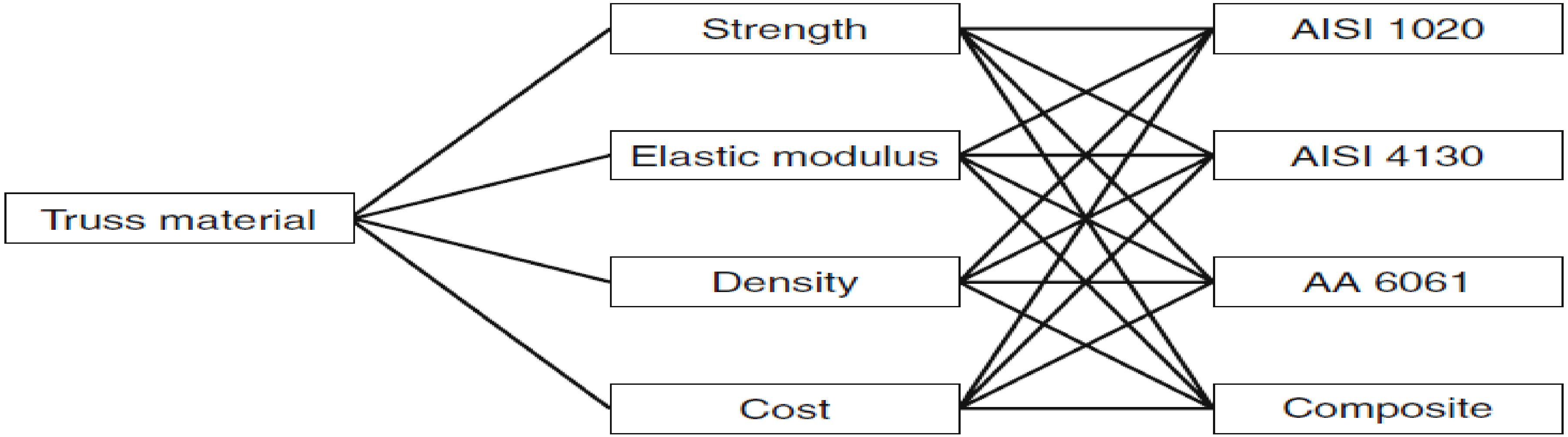
$$CR = \frac{CI}{RI}$$

If the value of Consistency Ratio is smaller or equal to 10%, the inconsistency is acceptable. If the Consistency Ratio is greater than 10%, we need to revise the subjective judgment.

A WORKED EXAMPLE

Materials Selection using AHP

by ABBAS KHAMMAS HUSSEIN



	Yield	Modulus	Density	Cost
AISI1020	280	210	7.8	5
AISI4130	1520	212	7.8	3
AA6061	275	70	2.7	4
EPoxyGlass	1270	28	2.1	2

Pairwise Comparison Matrix

	Yield	Modulus	Density	Cost
Yield	1	5	4	7
Modulus	$\frac{1}{5}$	1	$\frac{1}{2}$	3
Density	$\frac{1}{4}$	2	1	3
Cost	$\frac{1}{7}$	$\frac{1}{3}$	$\frac{1}{3}$	1

	Yield	Modulus	Density	Cost
Yield	1	5	4	7
Modulus	0.2	1	0.5	3
Density	0.25	2	1	3
Cost	0.14	0.33	0.33	1
Sum	1.59	8.33	5.83	14

Normalized Pairwise Comparison Matrix					
	Yield	Modulus	Density	Cost	**Criteria Weights**
Yield	0.629	0.600	0.686	0.500	0.604
Modulus	0.126	0.120	0.086	0.214	0.136
Density	0.157	0.240	0.172	0.214	0.196
Cost	0.088	0.040	0.057	0.071	0.064

Calculating The Consistency				
Criteria Weights	0.604	0.136	0.196	0.064
	Yield	Modulus	Density	Cost
Yield	1	5	4	7
Modulus	0.2	1	0.5	3
Density	0.25	2	1	3
Cost	0.14	0.33	0.33	1

Calculating The Consistency				
Criteria Weights	0.604	0.136	0.196	0.064
	Yield	Modulus	Density	Cost
Yield	1*0.604	5*0.136	4*0.196	7*0.064
Modulus	0.2*0.604	1*0.136	0.5*0.196	3*0.064
Density	0.25*0.604	2*0.136	1*0.196	3*0.064
Cost	0.14*0.604	0.33*0.136	0.33*0.196	1*0.064

Calculating The Consistency						
Criteria Weights	0.604	0.136	0.196	0.064		
	Yield	Modulus	Density	Cost	**Weighted Sum Value**	**Criteria Weights**
Yield	0.604	0.680	0.784	0.448	2.516	0.604
Modulus	0.121	0.136	0.098	0.192	0.547	0.136
Density	0.151	0.272	0.196	0.192	0.811	0.196
Cost	0.085	0.045	0.065	0.064	0.258	0.064

Weighted Sum Value(W2)	Criteria Weights(W1)	W2/W1
2.516	0.604	4.1655629
0.5468	0.136	4.0205882
0.811	0.196	4.1377551
0.25812	0.064	4.033125
	Average λ_{max}	4.0892578

$$Consistency\ Index\ (C.I) = \frac{\lambda_{max} - n}{1 - n}$$

$$= \frac{4.08 - 4}{1 - 4}$$

$$= 0.033\ \ , where\ n\ is\ the\ number\ of\ propertis.$$

$$Consistency\ Ratio = \frac{CI}{Random\ Index\ (R.I)} = \frac{0.033}{0.9}$$

$$= 0.037, \quad CR < 0.1\ , which\ mean\ our\ marix\ is\ consistent$$

Properties	Criteria Weights
Yield	0.604
Modulus	0.136
Density	0.196
Cost	0.064

	Yield	Modulus	Density	Cost	MRPI
AISI1020	280	210	7.8	5	199.5288
AISI4130	1520	212	7.8	3	948.6328
AA6061	275	70	2.7	4	176.4052
EPoxyGlass	1270	28	2.1	2	771.4276

Materials	MRPI	Rank	
AISI1020	199.5288	3	
AISI4130	948.6328	✓ 1	Best Material
AA6061	176.4052	4	
EPoxyGlass	771.4276	2	